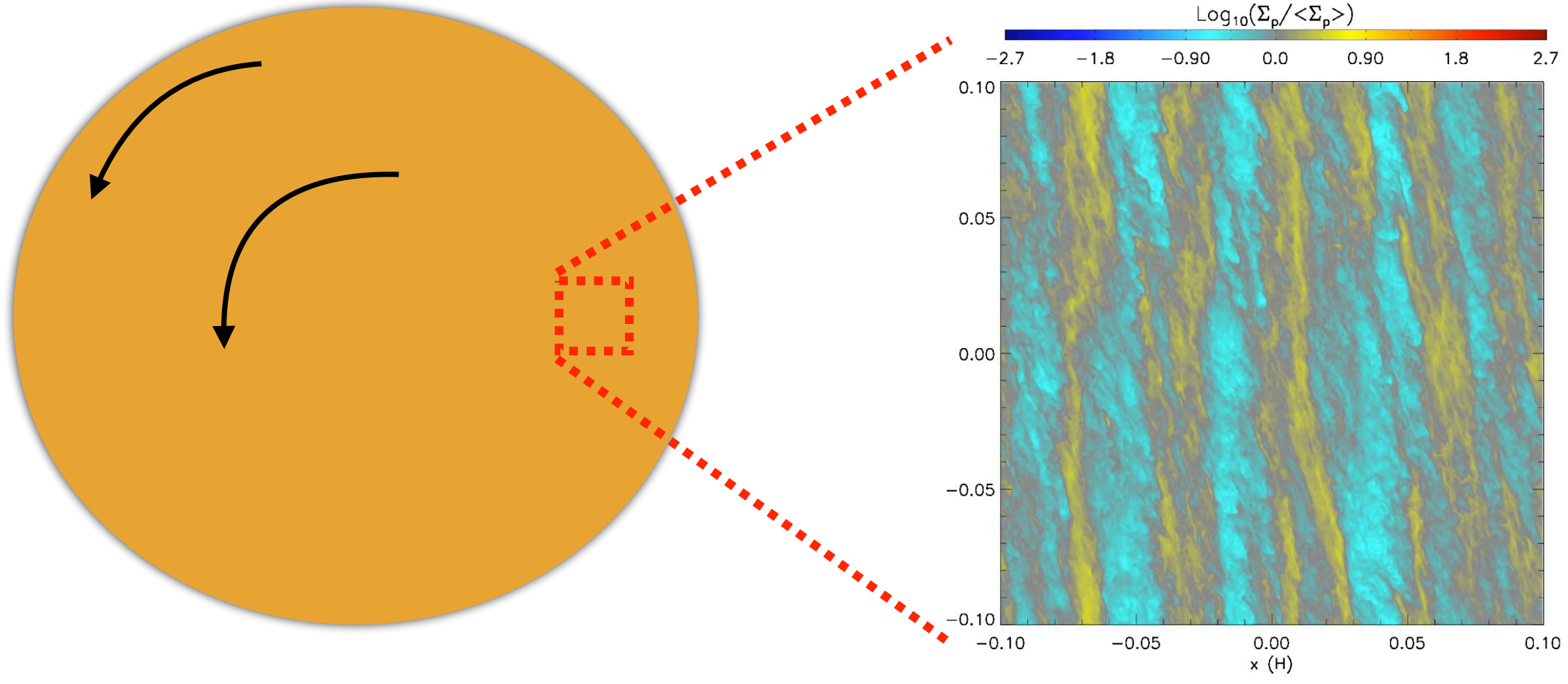


Planet Formation II

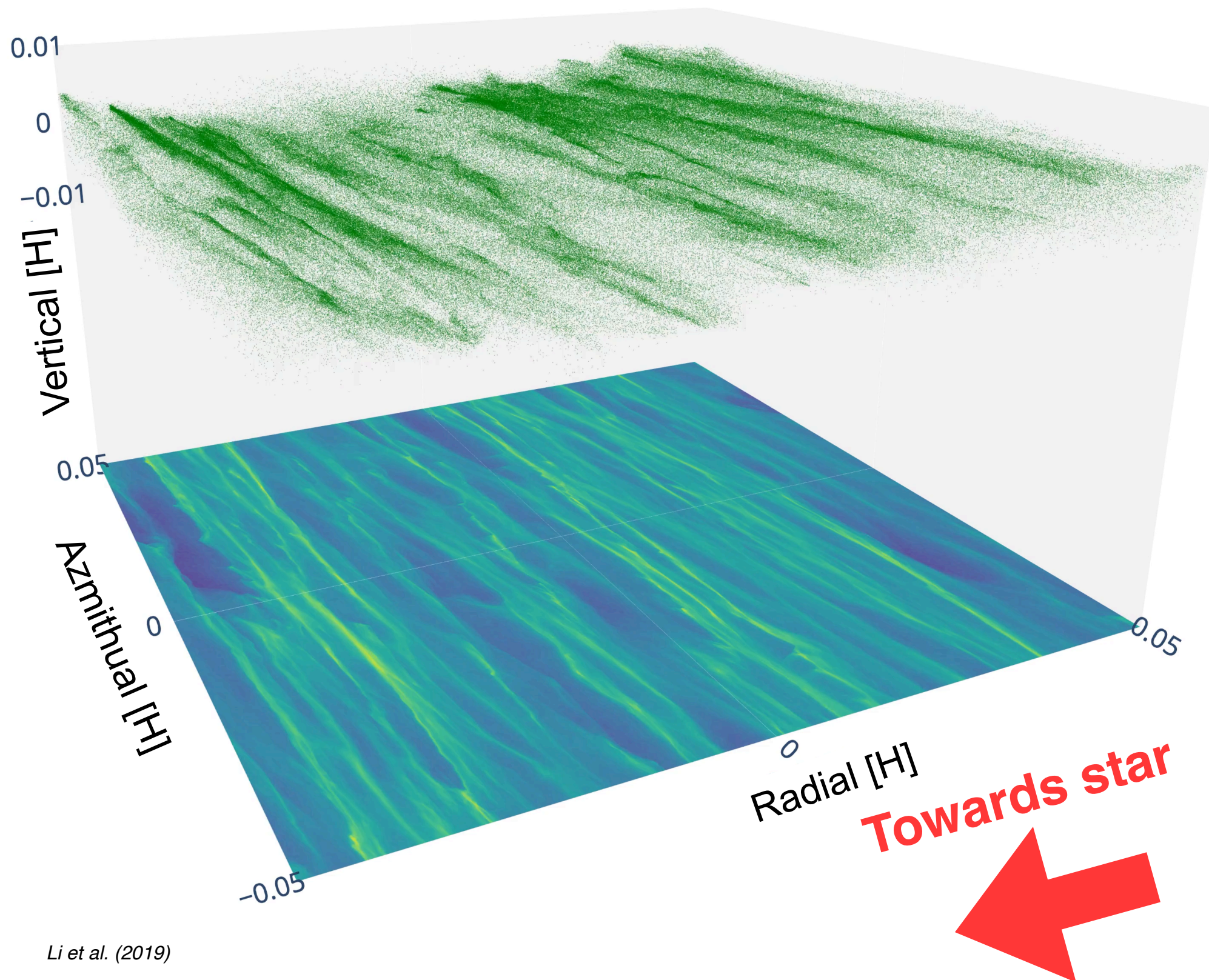
Jake Simon
Iowa State University

Streaming Instability

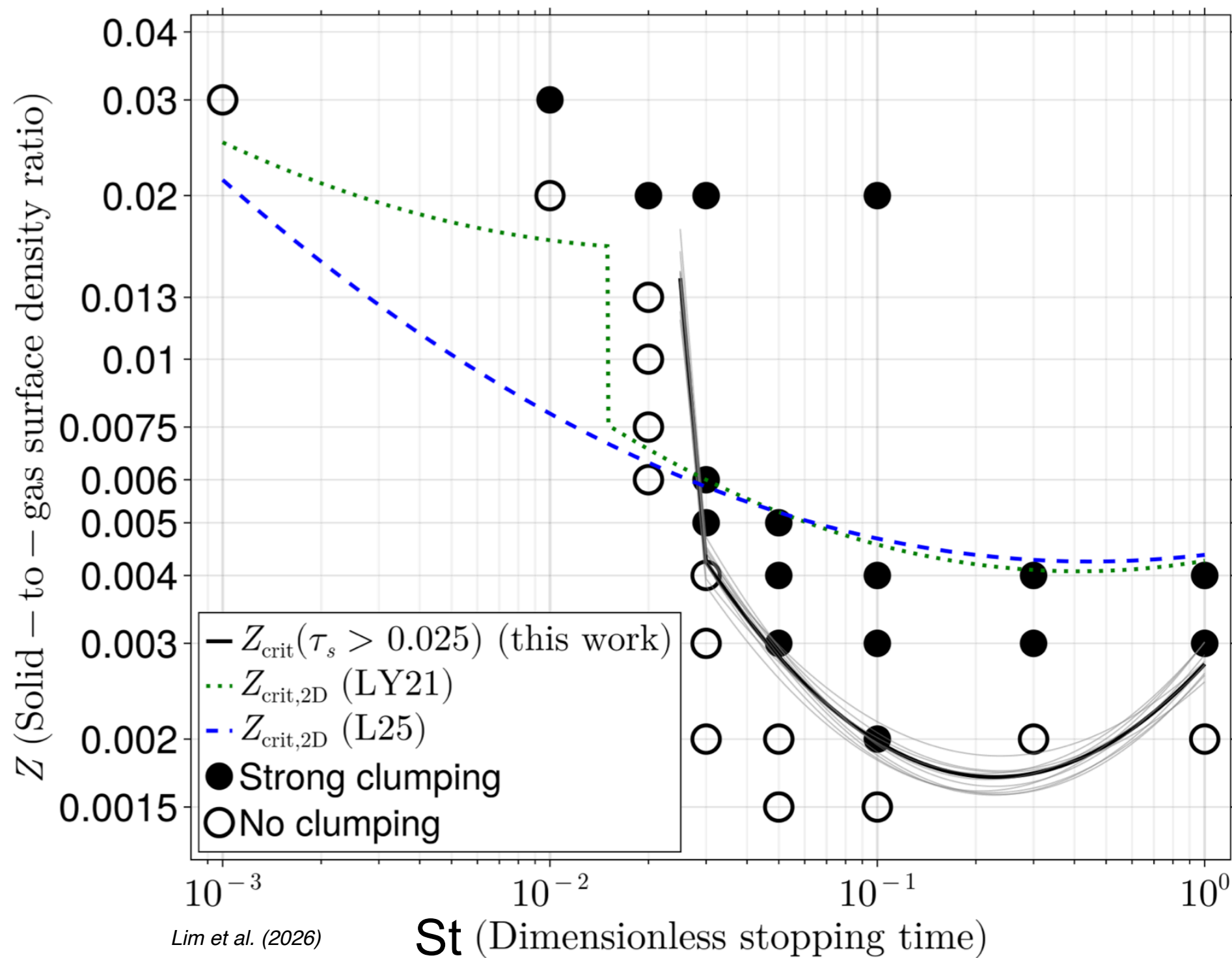


- When one accounts for not only gas drag on particles, but pushback on gas *from particles* a new process emerges: **the streaming instability**
- **Also, radial drift slows down!**
- **Streaming instability concentrates pebbles into filaments**
- Gravity eventually takes over; **collapse into ~100 km sized objects**

End result: gravitationally bound objects



Under what conditions does the streaming instability produce “strong clumping”?



- 2D and 3D critical curves differ
- 3D more expensive, but necessary
- Smaller St requires larger Z to produce strong clumping
- **VERY** difficult to deal with small St and large Z - equations become very stiff

What does stiff mean here?

$$\frac{\partial \rho_g}{\partial t} - q\Omega x \frac{\partial \rho_g}{\partial y} + \mathbf{u} \cdot \nabla \rho_g + \rho_g \nabla \cdot \mathbf{u} = 0,$$

Gas continuity eqn.

$$\begin{aligned} \frac{\partial \mathbf{u}}{\partial t} - q\Omega x \frac{\partial \mathbf{u}}{\partial y} + \mathbf{u} \cdot \nabla \mathbf{u} = & a_x \hat{\mathbf{e}}_x + g_z(z_k) \hat{\mathbf{e}}_z - c_s^2 \nabla \ln \rho_g \\ & - 2\boldsymbol{\Omega} \times \mathbf{u} + q\Omega u_x \hat{\mathbf{e}}_y + \sum_j \frac{\tilde{\epsilon}_{kj}(\mathbf{v}_j - \mathbf{u})}{t_s}, \end{aligned}$$

Gas momentum eqn.

$$\frac{d\mathbf{r}_{p,j}}{dt} = -q\Omega x_{p,j} \hat{\mathbf{e}}_y + \mathbf{v}_j,$$

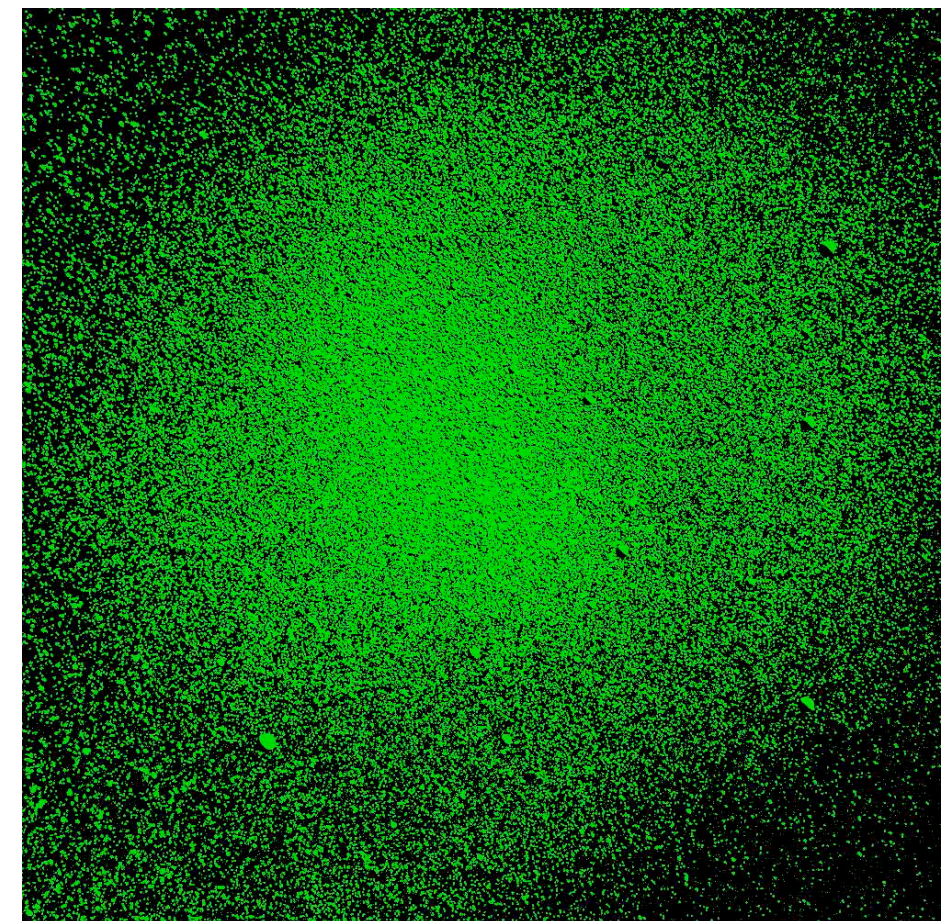
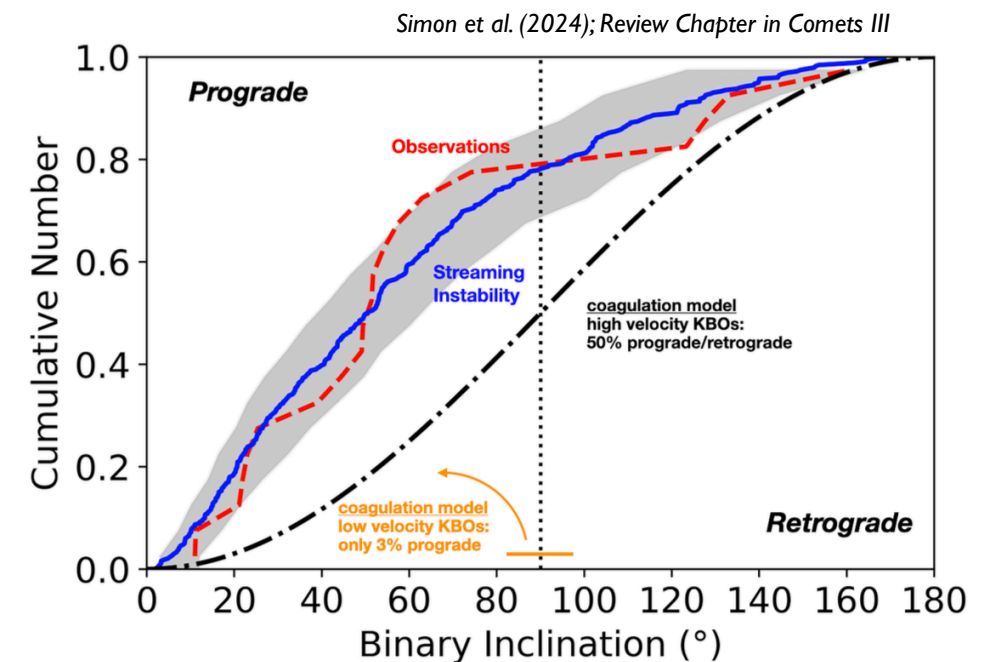
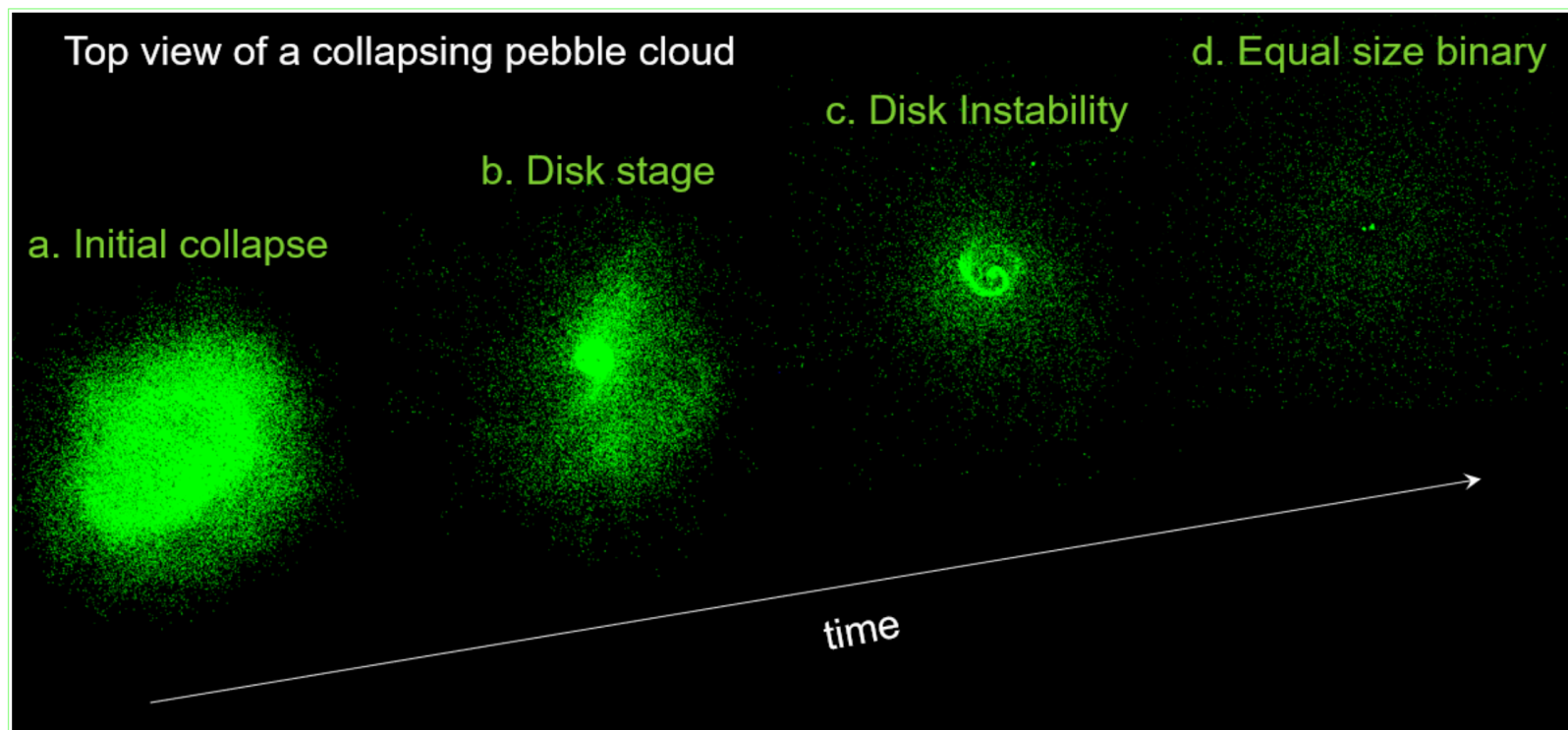
Particle eqns of motion

$$\frac{d\mathbf{v}_j}{dt} = g_z(z_{p,j}) \hat{\mathbf{e}}_z - 2\boldsymbol{\Omega} \times \mathbf{v}_j + q\Omega v_{j,x} \hat{\mathbf{e}}_y + \frac{\tilde{\mathbf{u}}_j - \mathbf{v}_j}{t_s}.$$

Stiffness condition:

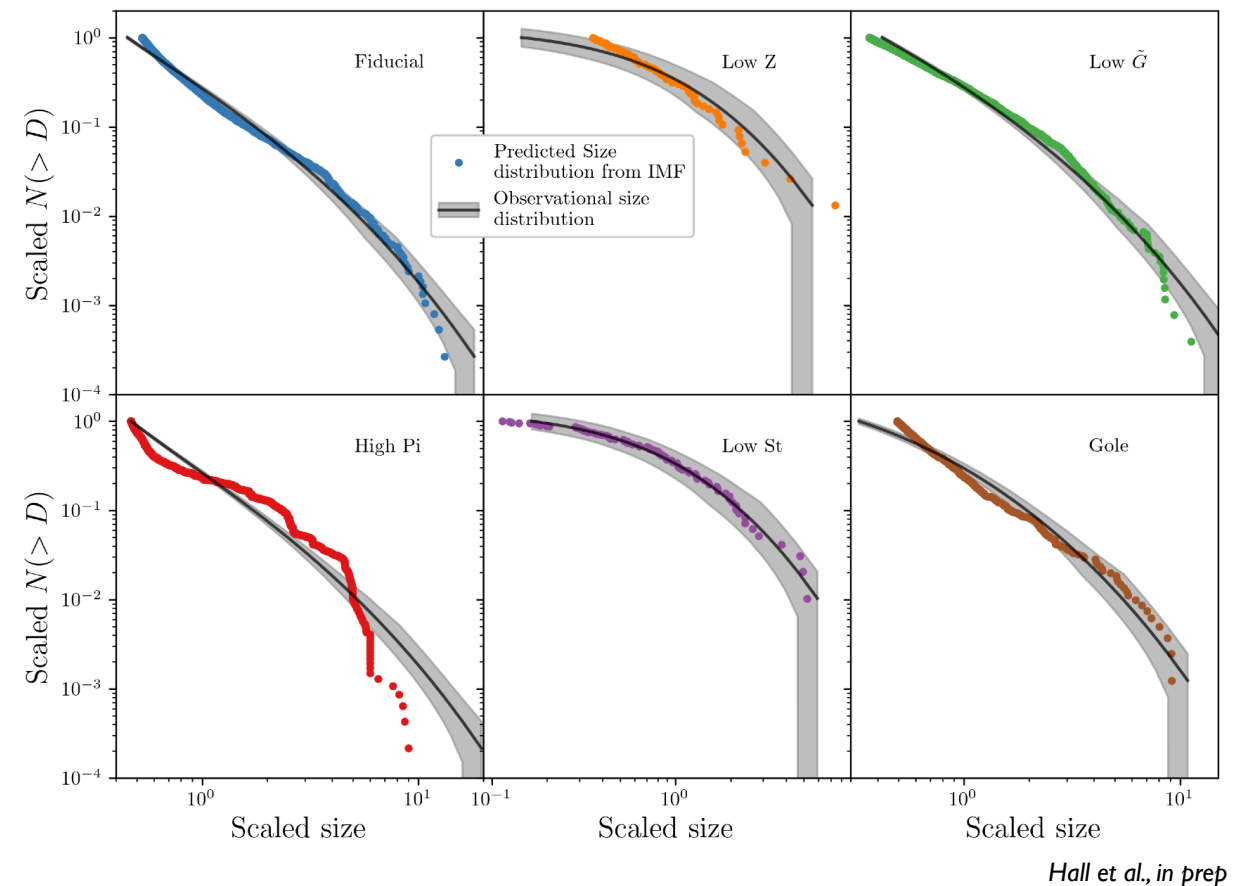
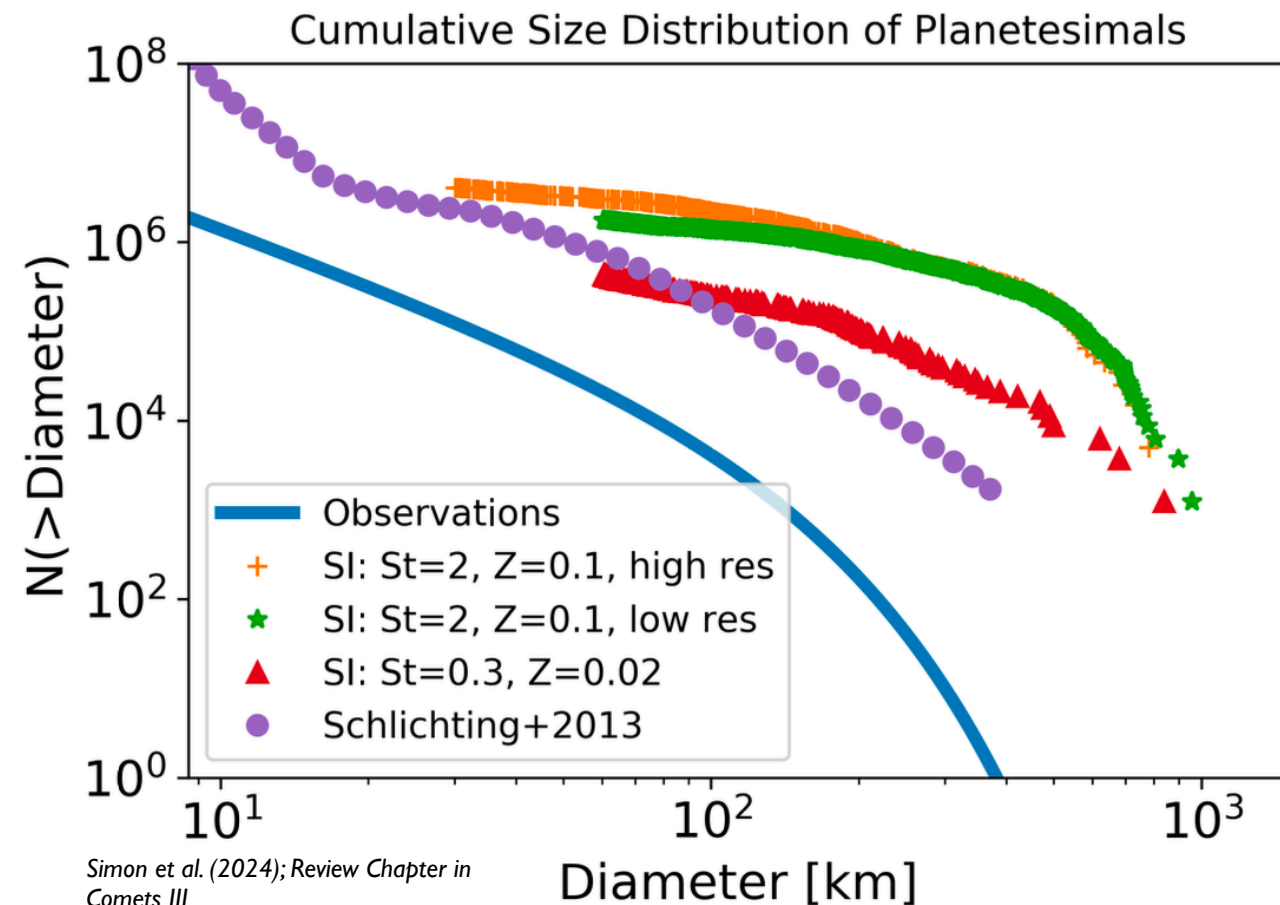
$$\Delta t > \frac{t_s}{1 + \epsilon}$$

Tests of the Streaming Instability: Binary KBO properties



- Pebble clouds initially have a lot of angular momentum
- **Leads to binary formation** (too much spin for a single body to stay bound)
- Binary properties **strongly agree with Kuiper Belt binaries!**

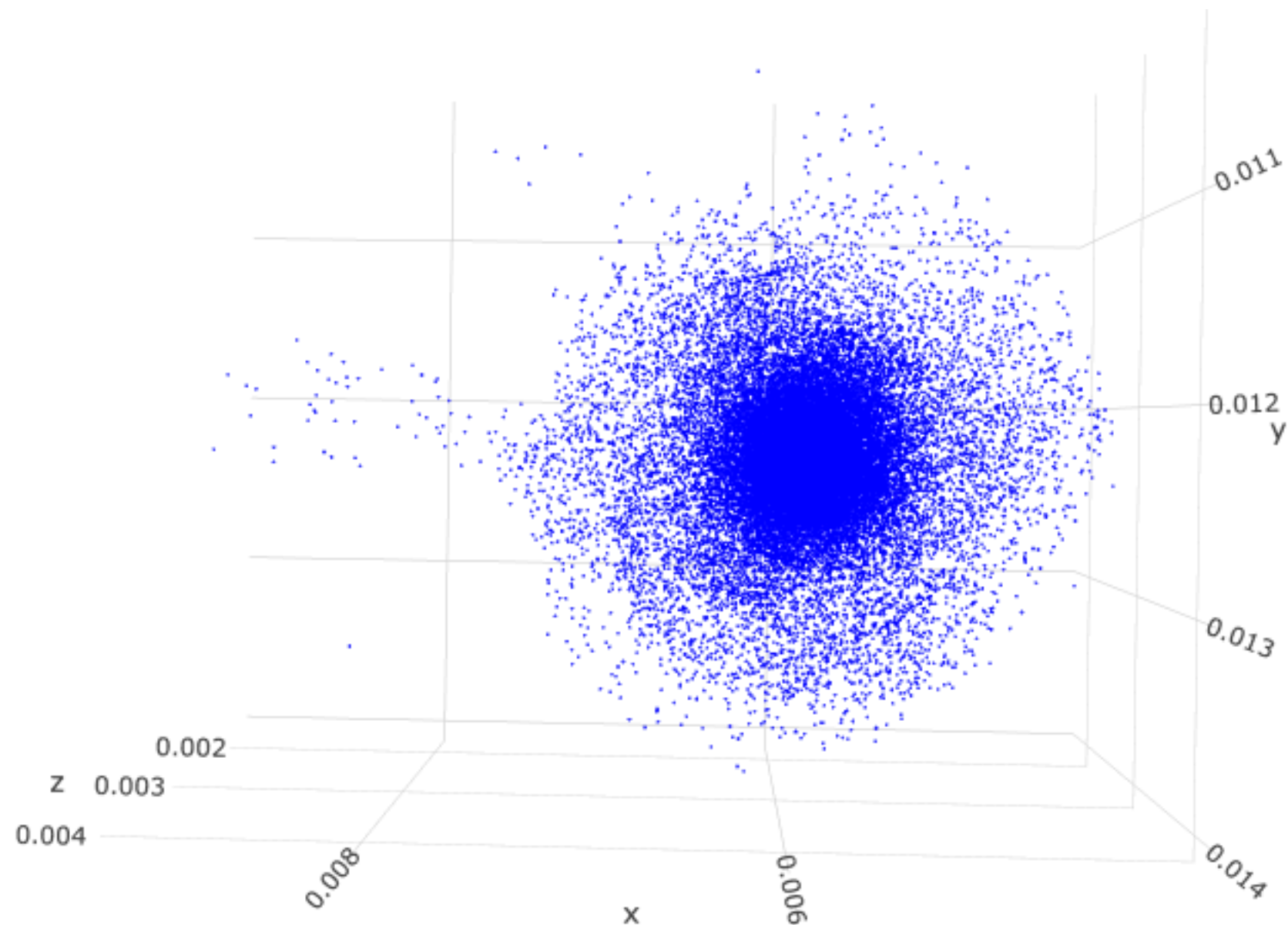
Tests of the Streaming Instability: Size Distribution



- Compare with cold classical Kuiper Belt (pristine!)
- Earlier work: Shape of (cumulative) size distribution doesn't agree
- Newer work (by my graduate student Weston Hall) - **shapes strongly agree!!!**
- **But still issues to resolve:**
 - Can only test large-size end of distribution ($D \gtrsim 50$ km)
 - Very idealized setups (no MHD, no self-consistent substructures)
 - “Planetesimals” are actually uncollapsed pebble clouds
 - Stiffness issue - what happens at small St and large Z ??

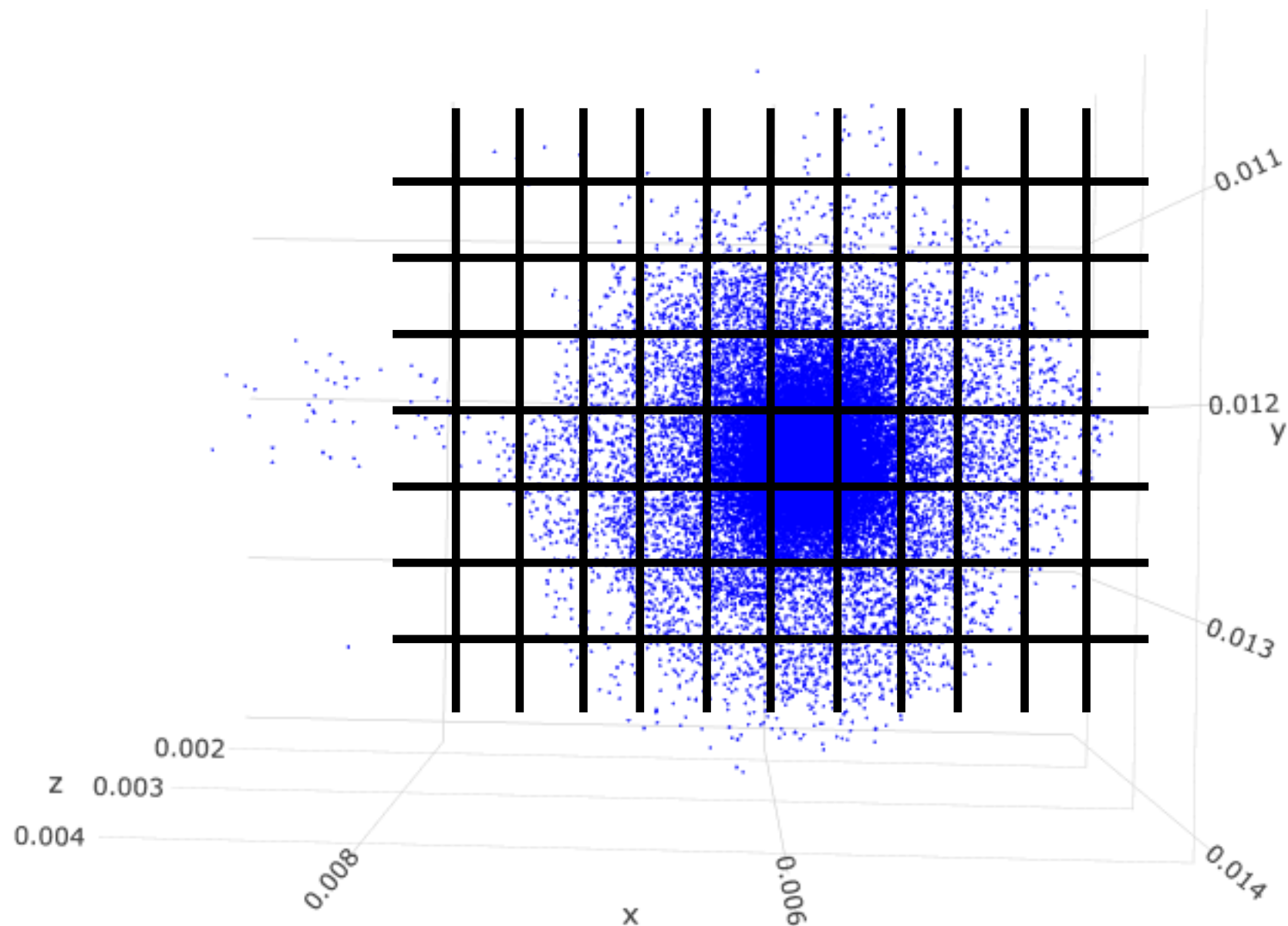
Uncollapsed pebble cloud limited by grid scale

(Poisson solver only resolves gravity on grid scale)



Uncollapsed pebble cloud limited by grid scale

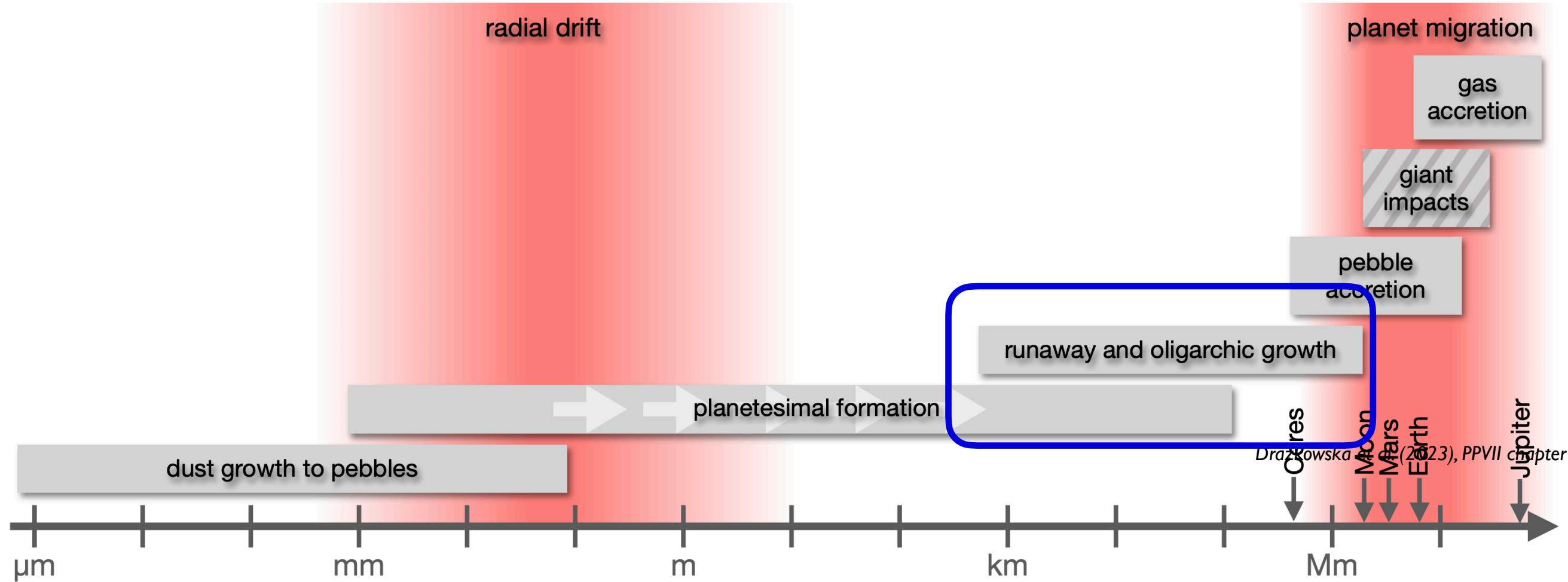
(Poisson solver only resolves gravity on grid scale)



AthenaK Plans

1. Simplest problem to address: explore small St and large Z
(We're putting in a new particle integrator, either using implicit or analytic approach)
2. Larger shearing boxes with MHD turbulence, non-ideal effects (diffusion, Hall), substructures (zonal flows) and influence on the streaming instability
(takes advantage of GPU capability)
3. *Use AMR* to resolve pebble cloud collapse (need at least 11 levels of refinement)
4. *Use AMR* to probe small size end of the distribution (only requires ~ 7 levels of refinement)

Next stage is embryo formation



What is a planetary embryo?



- **Planetary embryos** (sometimes called **protoplanets**) are bodies between the size of planetesimals and Earth-sized objects. So:
 $\sim 10^3 - 10^4$ km
- They are the phase between planetesimals and fully grown planets
- Mars, even though classified as a planet, can be considered a **stranded embryo** (actually part of something called the small Mars problem)

Two modes of forming planetary embryos from planetesimals

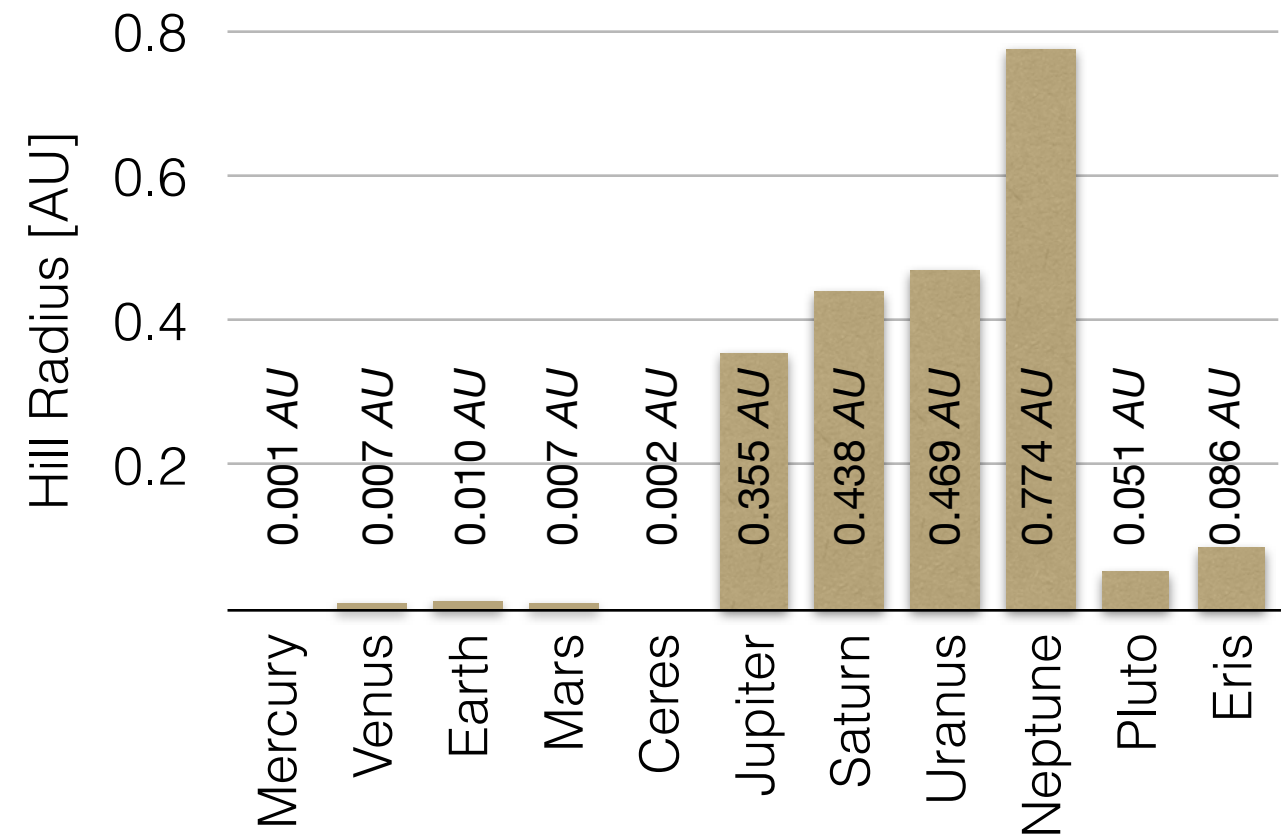
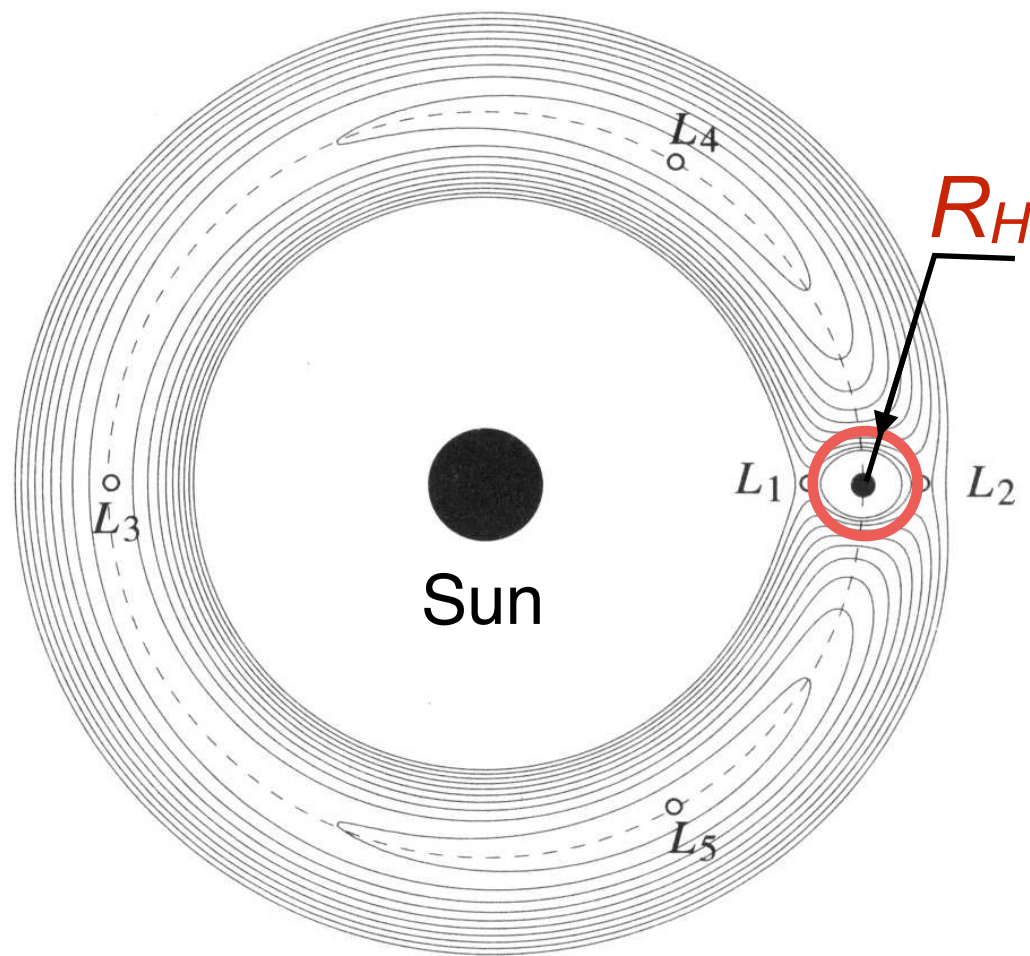
- **Planetesimal accretion**

- Planetesimals merge and accrete to form larger objects
- Slow process - difficult to produce planets before gas disappears (necessary for giant planets to have atmospheres)
- Depends on gravitational focusing (related to Hill Radius)

- **Pebble accretion**

- Gas drag assisted accretion of remaining mm-cm pebbles onto planetesimals
- MUCH more rapid than planetesimal accretion

Hill Radius

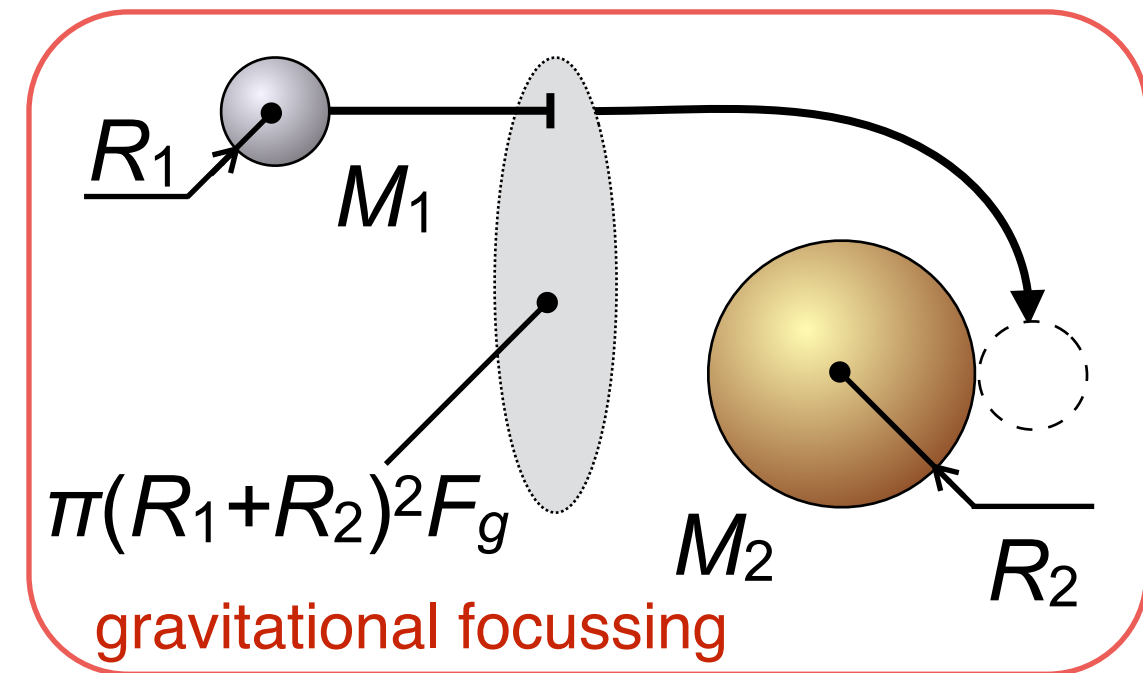
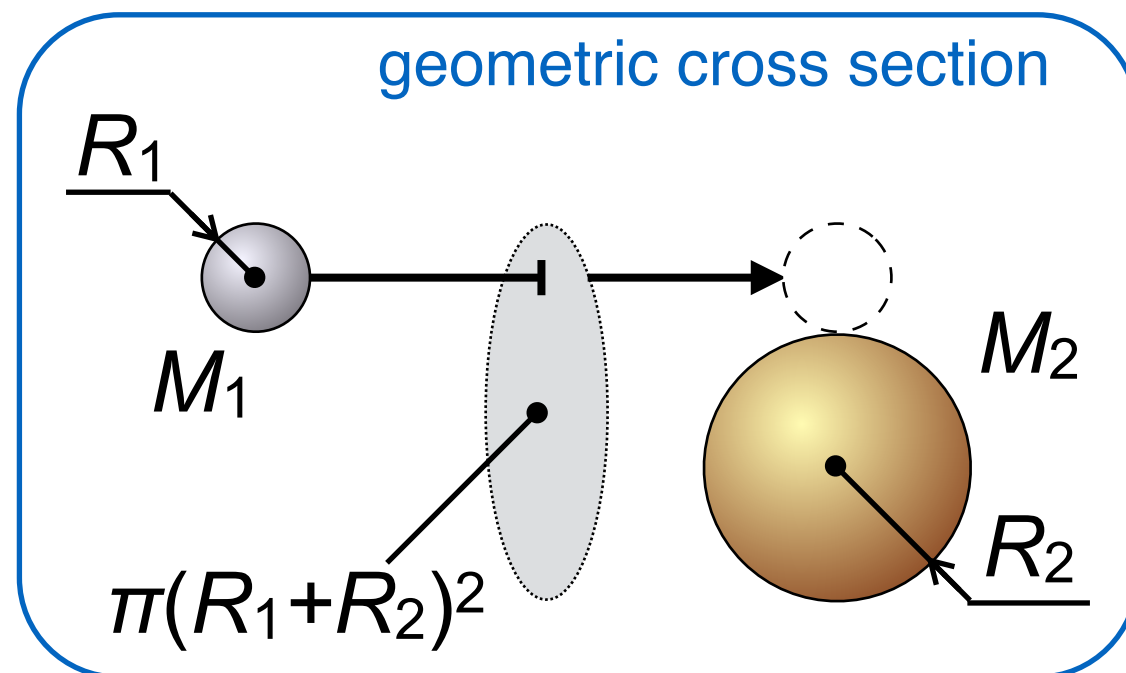


- The **Hill Sphere** is the region where the gravity of an orbiting body is dominant with respect to the gravity of the star. The **Hill Radius** is:

$$R_H = \left(\frac{M_P}{3M_*} \right)^{1/3} a$$

- Objects passing through the Hill Radius of a planetesimal are accreted.
- Very closely related to the Roche limit.

Gravitational Focusing



- If mutual gravity is not a factor, the collisional cross section between two bodies should just be the area defined by the sum of their radii.
- Gravity however brings together objects further away from that, by bending their trajectory due to mutual attraction. The collisional cross section is increased by a **focusing factor** F_g so that:

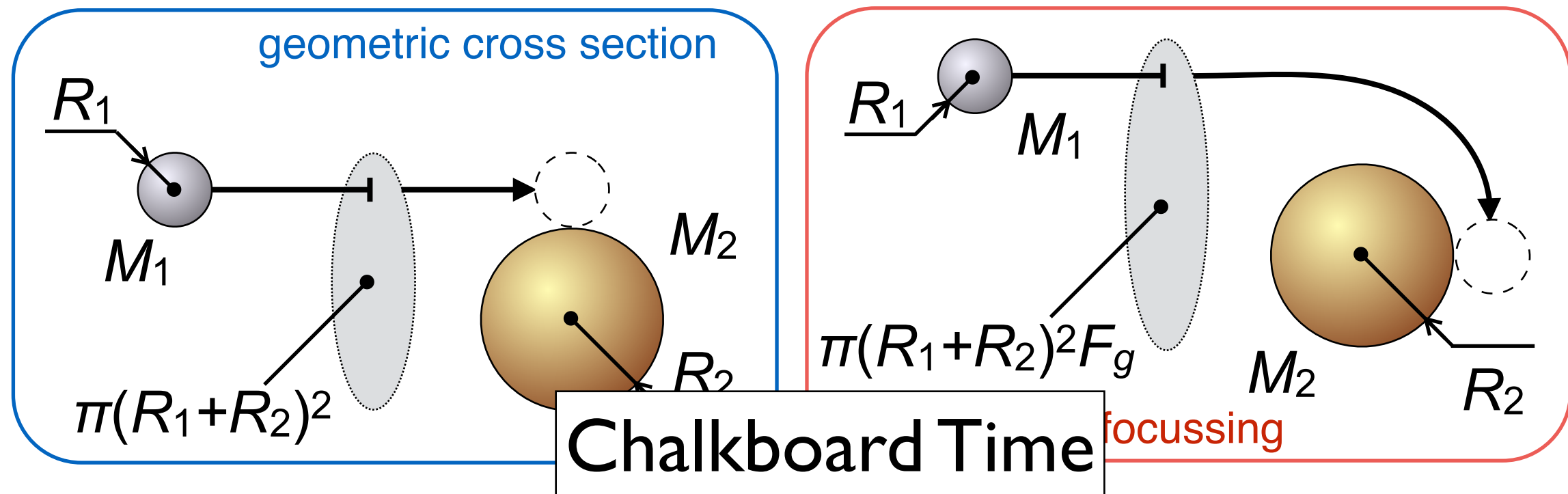
$$\sigma_{accr} = \pi (R_1 + R_2)^2 F_g \quad \text{with} \quad F_g = 1 + \left(\frac{v_{esc}}{v_{\infty}} \right)^2$$

Relative velocity before mutual attraction $\xrightarrow{\text{red arrow}}$ v_{∞}

Escape velocity at point of contact $\xrightarrow{\text{blue arrow}}$ v_{esc}

$$v_{esc} = \sqrt{\frac{2G(M_1 + M_2)}{R_1 + R_2}}$$

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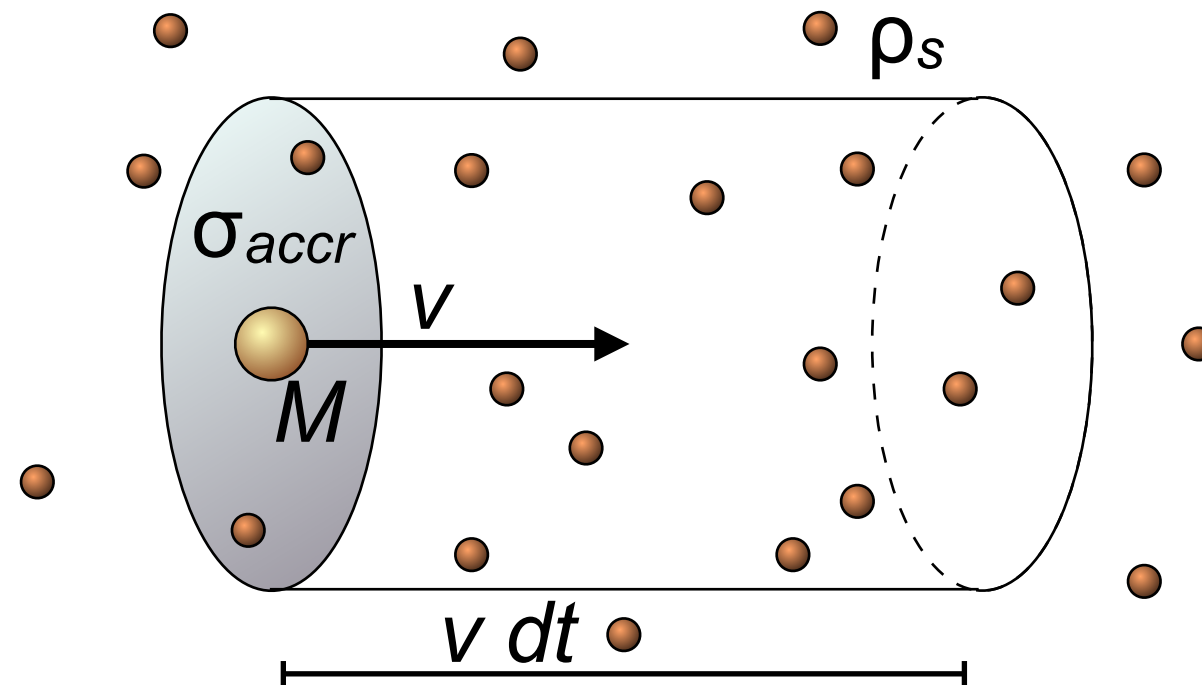
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Escape velocity at point of contact

Relative velocity before mutual attraction

Mass Growth Rate

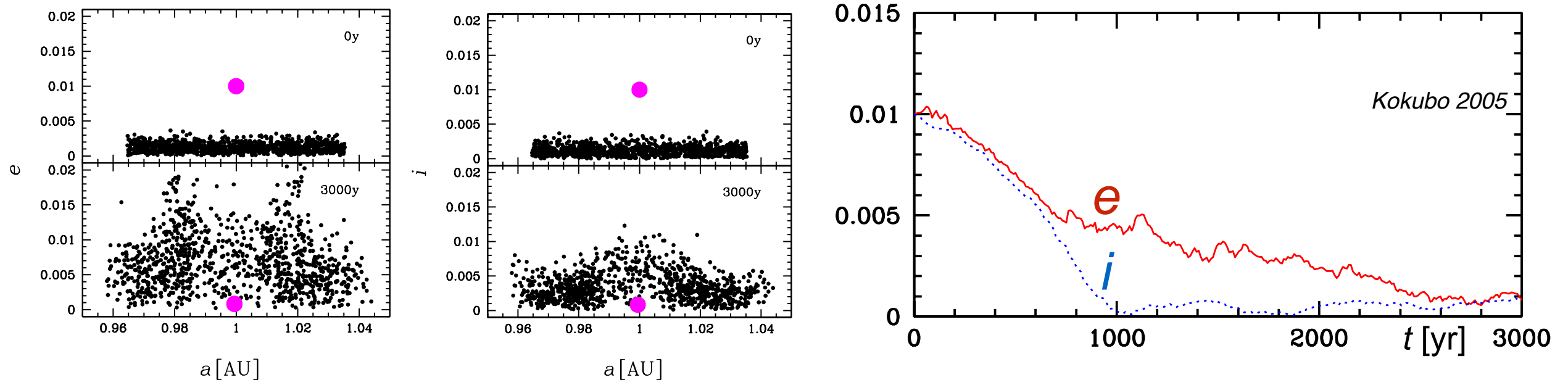


- The planetary embryos accrete at the expense of the smaller planetesimals swarming by them. The **accretion rate** is:

$$\frac{dM}{dt} = \rho_s v \pi R^2 F_g$$

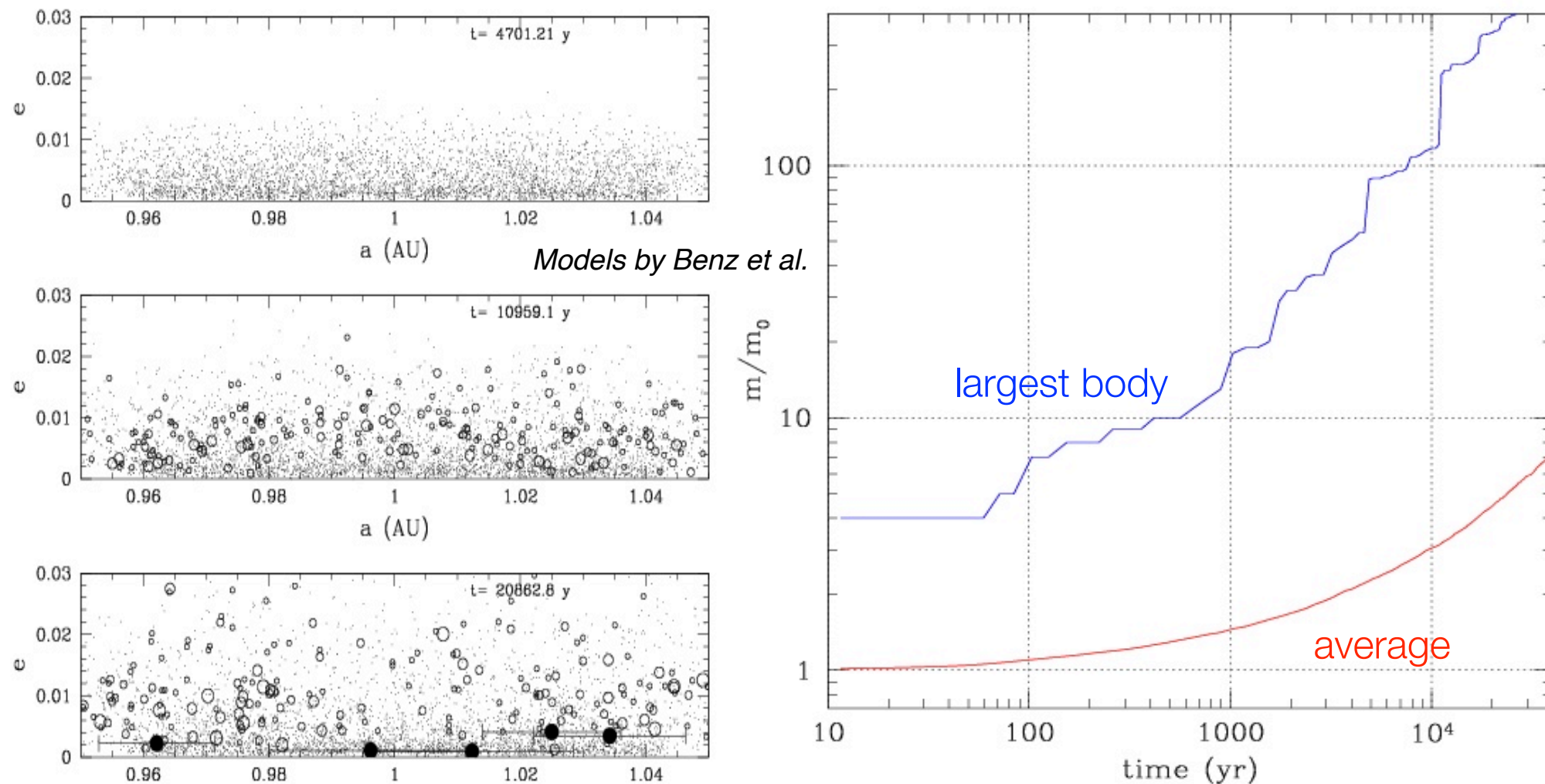
where ρ_s is the mass density of the planetesimals, v is their relative velocity with the embryos, and $\pi R^2 F_g$ is the accretion cross section (the cross section of the embryos $\times$ the enhancement factor).

Stirring and Damping Processes



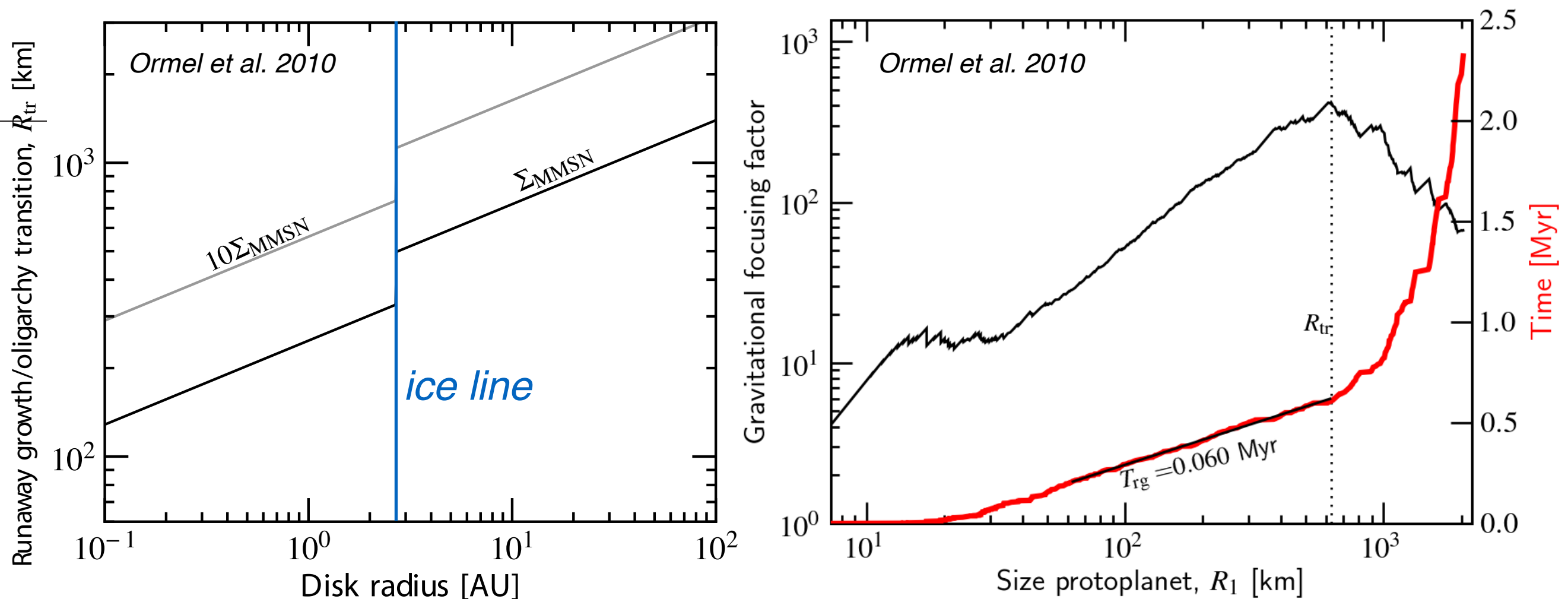
- Embryos affect the smaller planetesimals even if they don't accrete them: The eccentricity and inclination of the planetesimals are increased. The planetesimals are thus **stirred through dynamical encounters**
- Increase in velocity of planetesimals is however limited by **gas drag**.

Runaway Growth



- The system starts with a few embryos with circular coplanar orbits with larger mass ($\Rightarrow$ large v_{esc}) than surrounding planetesimals.
- Large $v_{\text{esc}} \rightarrow F_g \gg 1 \rightarrow F_g \approx (v_{\text{esc}}/v_{\infty})^2 \sim 2GM/(Rv_{\infty}^2) \propto R^3/R \sim R^2$
- Accretion rate $dM/dt \propto R^2 F_g \sim R^4$. Large protoplanets grow much faster than smaller planetesimals: **runaway growth** phase.

Oligarchic Growth



- Models shows that once the embryos grow above a critical radius, stirring increases velocities of planetesimals. This increases v_∞
- Large $v_\infty \rightarrow 1 + (v_e/v_\infty)^2 \sim \text{const} \rightarrow dM/dt \propto R^2 F_g \sim R^2 \sim M^{2/3}$.
- Larger protoplanetary embryos' growth slows down: **oligarchic growth**.

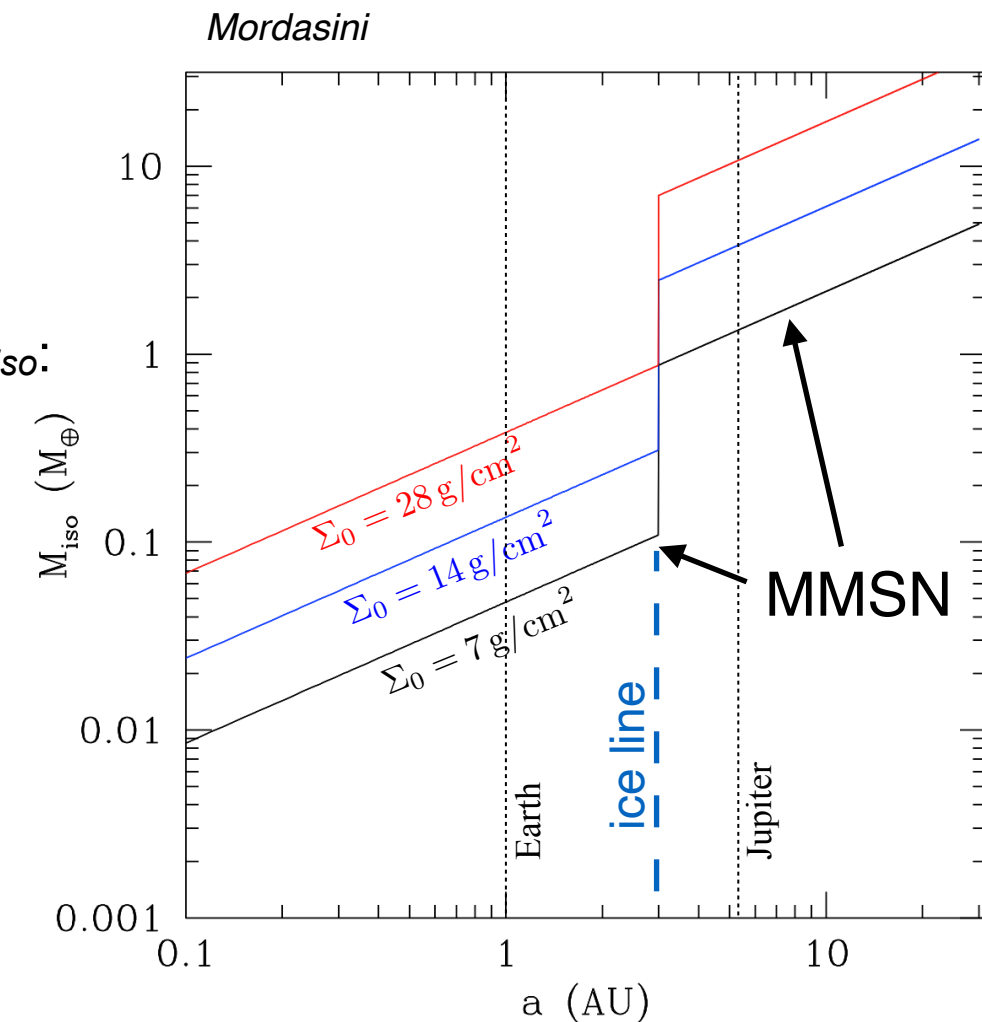
Isolation Mass and Orderly Growth

- During runaway and oligarchic growth all planetesimals with orbits within $\sim 4 R_H$ to a embryo will be accreted.
- At that point the embryo will run out of planetesimals and oligarchic growth will stop.
- Maximum mass accreted in this way is called **isolation mass** M_{iso} :

$$M_{iso} \approx 5 \times 10^{20} \left(a^2 \Sigma_\rho \right)^{3/2} M_*^{-1/2}$$

where M_{iso} is in kg, M_* is in $M_\odot$, a is in AU and the surface mass density of the disk Σ_ρ is in kg/m². Once M_{iso} is reached, embryos grow at a much slower rate: **orderly growth**.

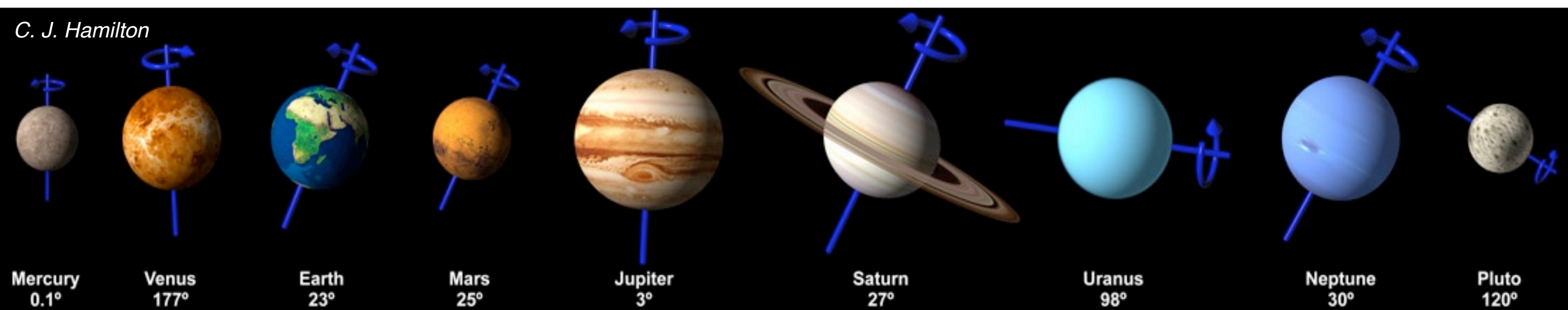
- For the minimum mass solar nebula (MMSN) $M_{iso} \approx 0.05 M_\oplus$ at 1 AU and $\approx 1.4 M_\oplus$ at 5.2 AU: cannot form cores of giant planets sufficiently close to the star.



Physical interpretation

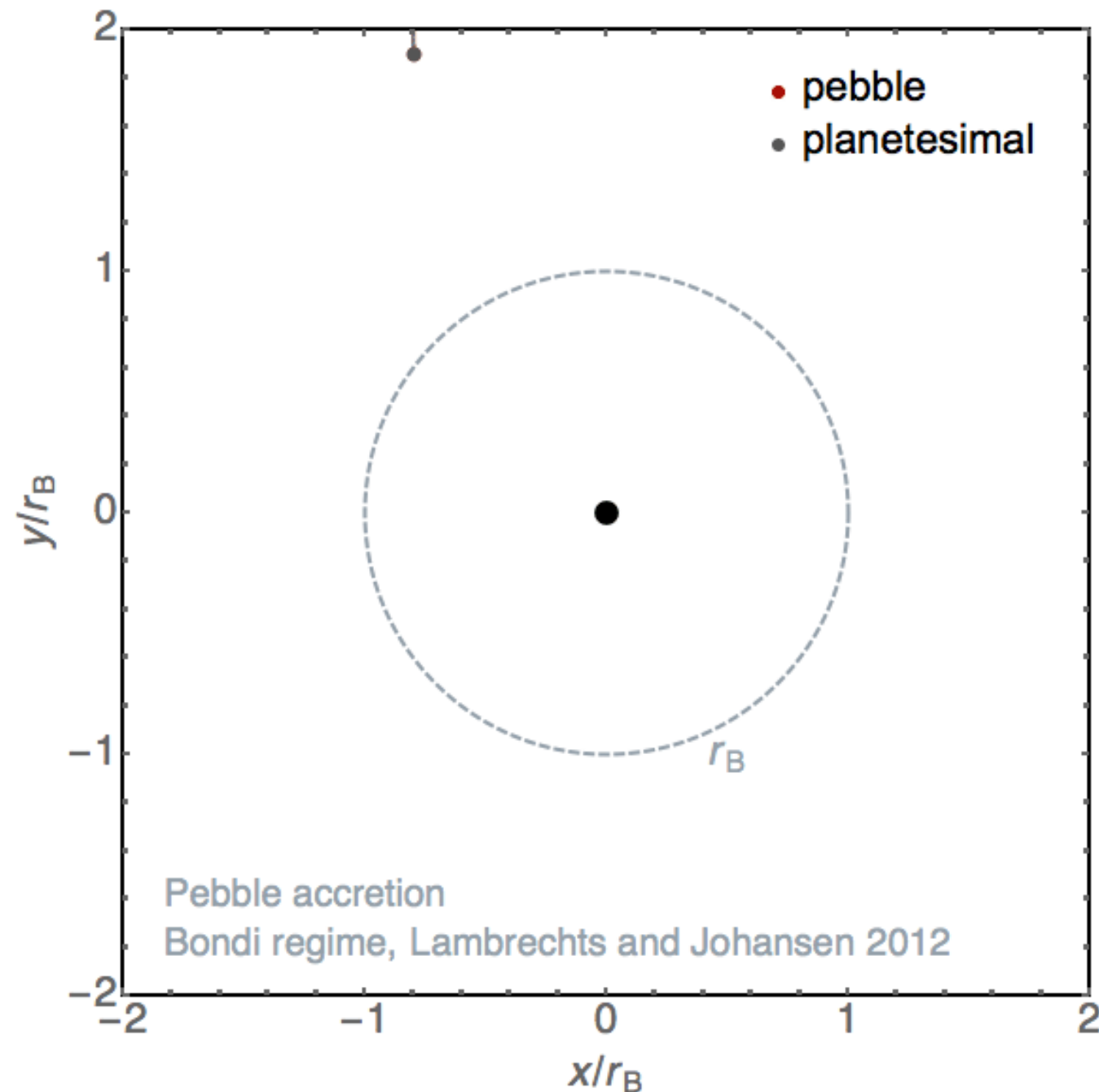
- **Higher solid surface density Σ_p** $\rightarrow$ more mass in the feeding zone $\rightarrow$ larger isolation mass.
- **Larger orbital radius a** $\rightarrow$ feeding zone is physically larger, so M_{iso} increases with a^3 .
- **More massive central star** $\rightarrow$ stronger gravity relative to embryo, smaller Hill sphere $\rightarrow$ smaller isolation mass.

Growth by Giant Impacts



- Once isolation mass is reached, planetary growth is driven by ***giant impacts*** between embryos and remaining planetesimals.
- Giant planets accrete gas envelopes very fast before disk gas depletion.
- Final growth of terrestrial planets is driven by gravitational perturbation of already formed giants.
- A phase with giant impacts necessary to explain obliquity (tilt) of some planets (e.g. Uranus), anomalous density of Mercury, certain moons.

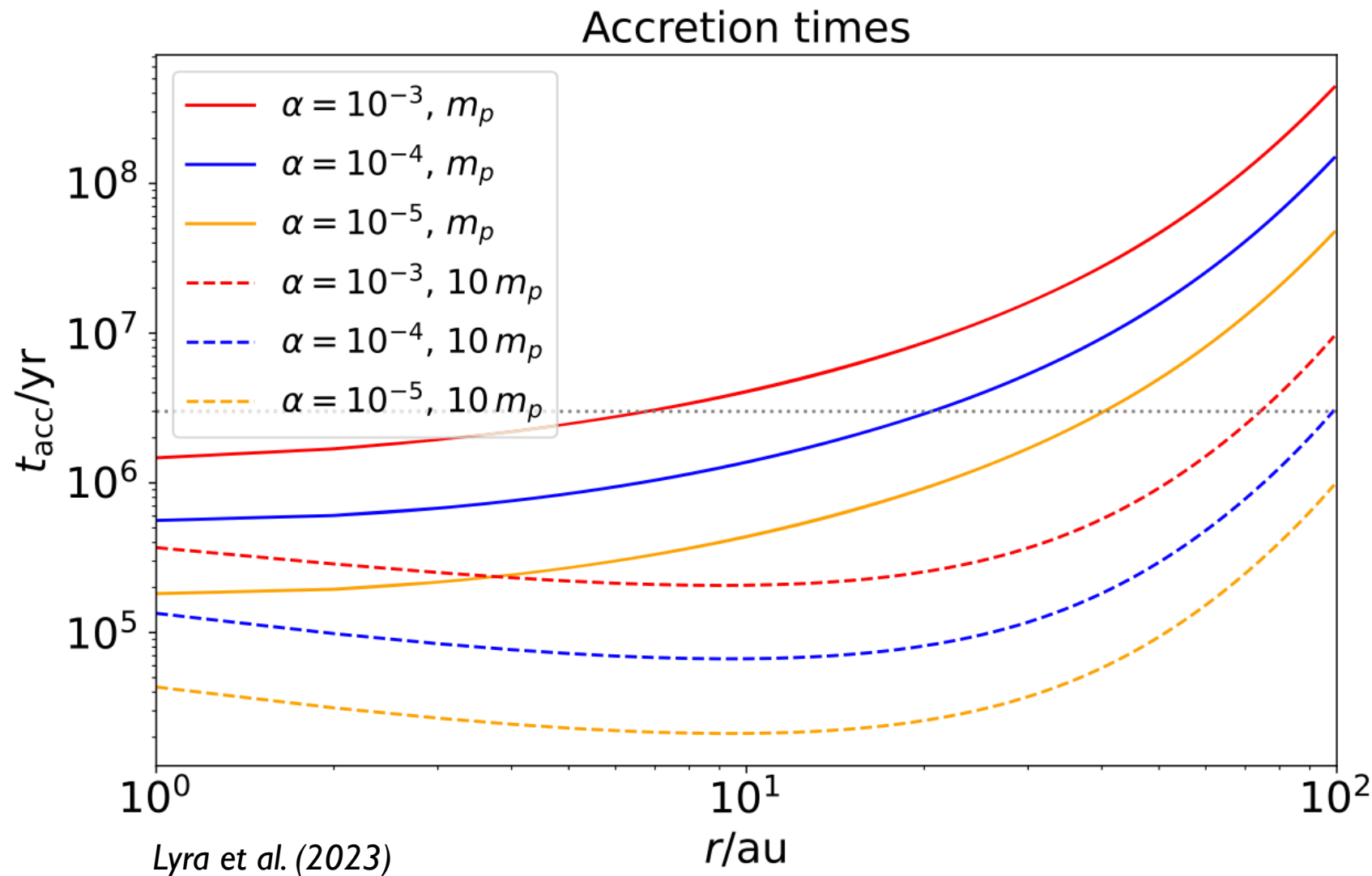
Pebble Accretion



- Gas drag removes momentum from particles, allowing them to spiral inward onto embryos; ***drastically increases the cross section***
- While you may need an embryo of some scale to already have formed, work by Lyra et al. 2023 **suggests that pebble accretion can work even for planetesimals!**

If true, we may not even need any embryos to form first!

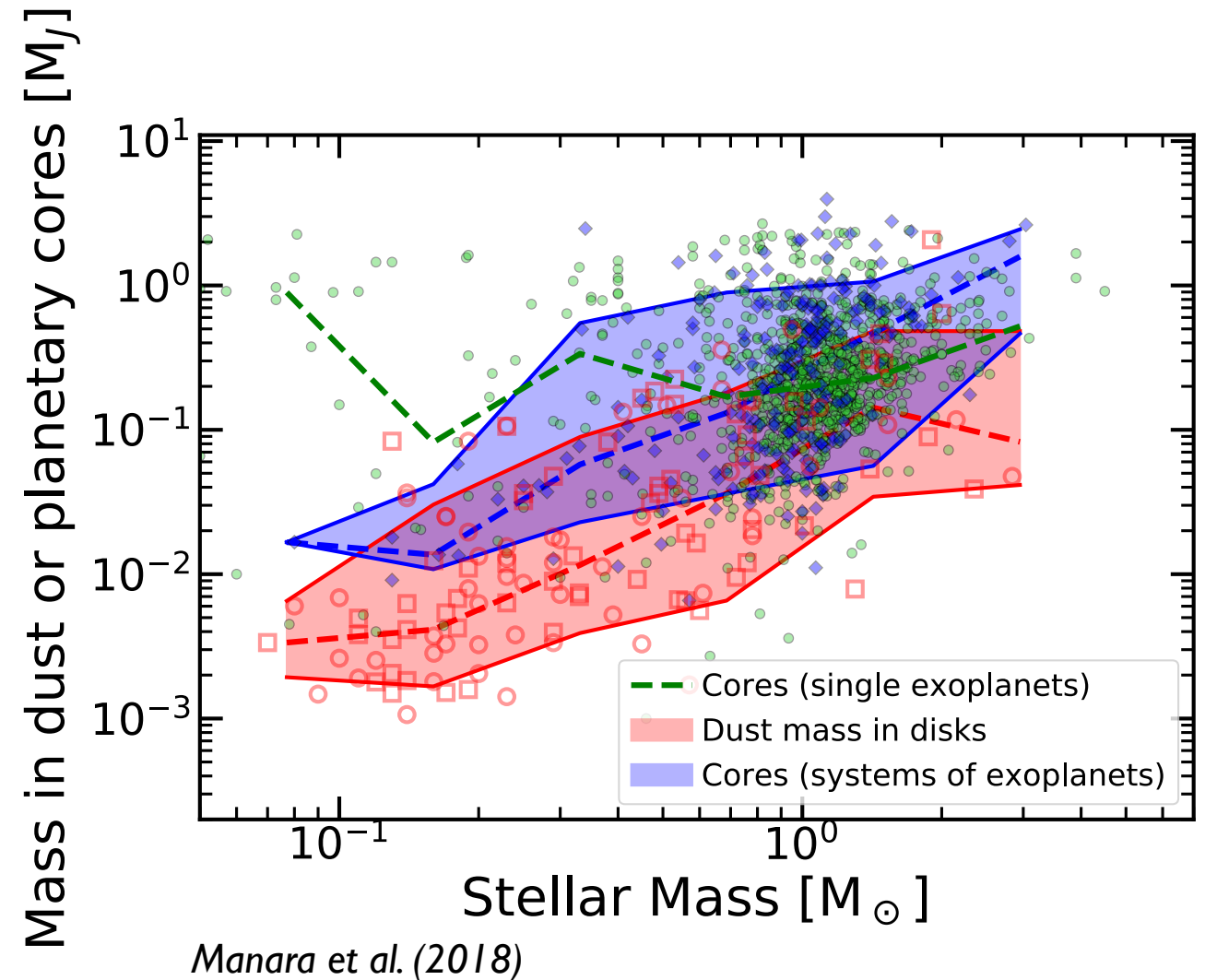
Pebble Accretion



- If you start with planetesimal masses, it can still take > 1 Myr to form larger bodies
- But if conditions are right, can form planets well before the gas dissipates (which happens at $\sim$ Myr) even with planetesimal masses

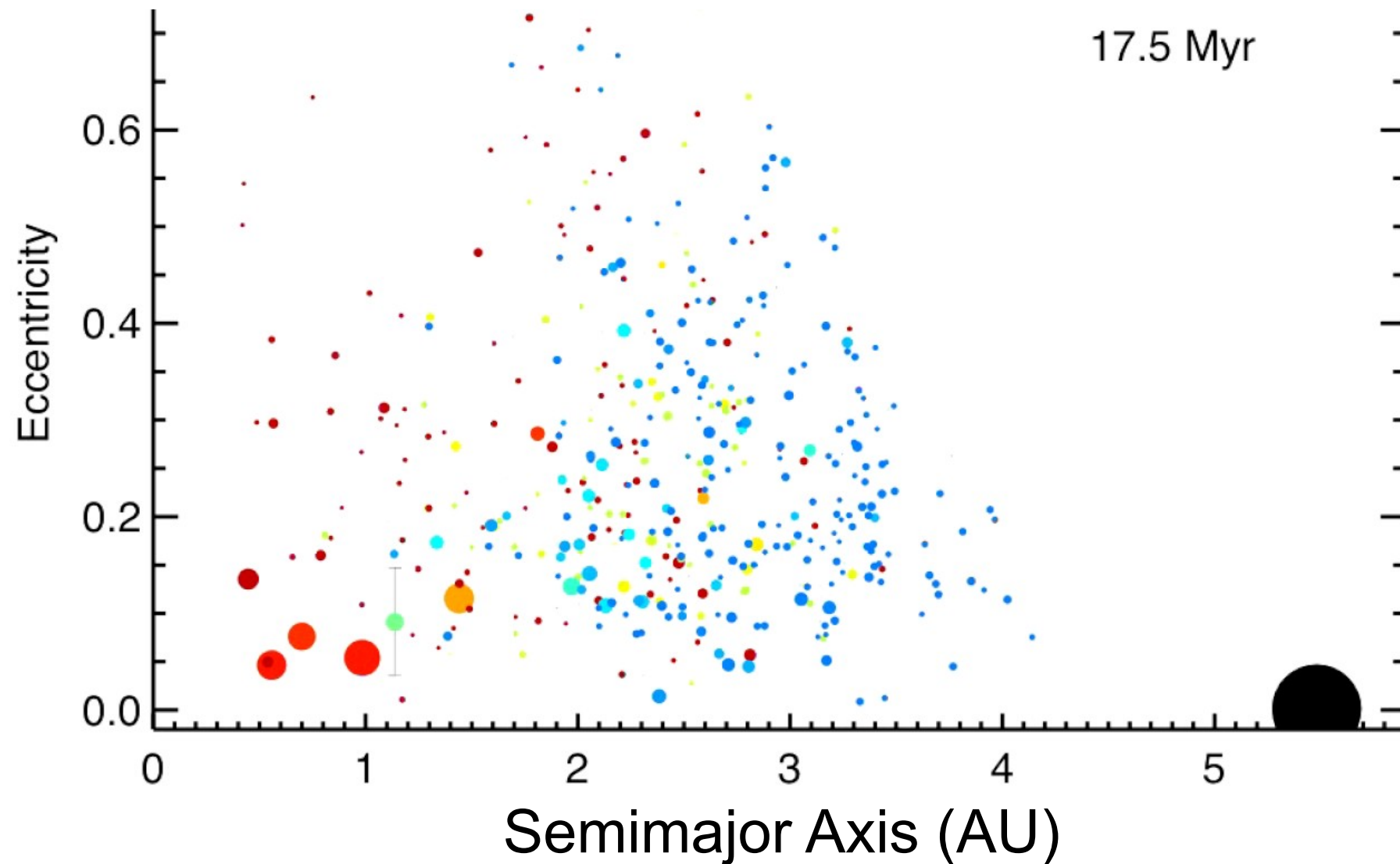
- **Pebble accretion is significantly more rapid than planetesimal accretion**
- Lots of subtleties, but the details depend largely on the embryo mass and the disk properties, degree of turbulence, etc.

But maybe pebble accretion isn't fast enough!



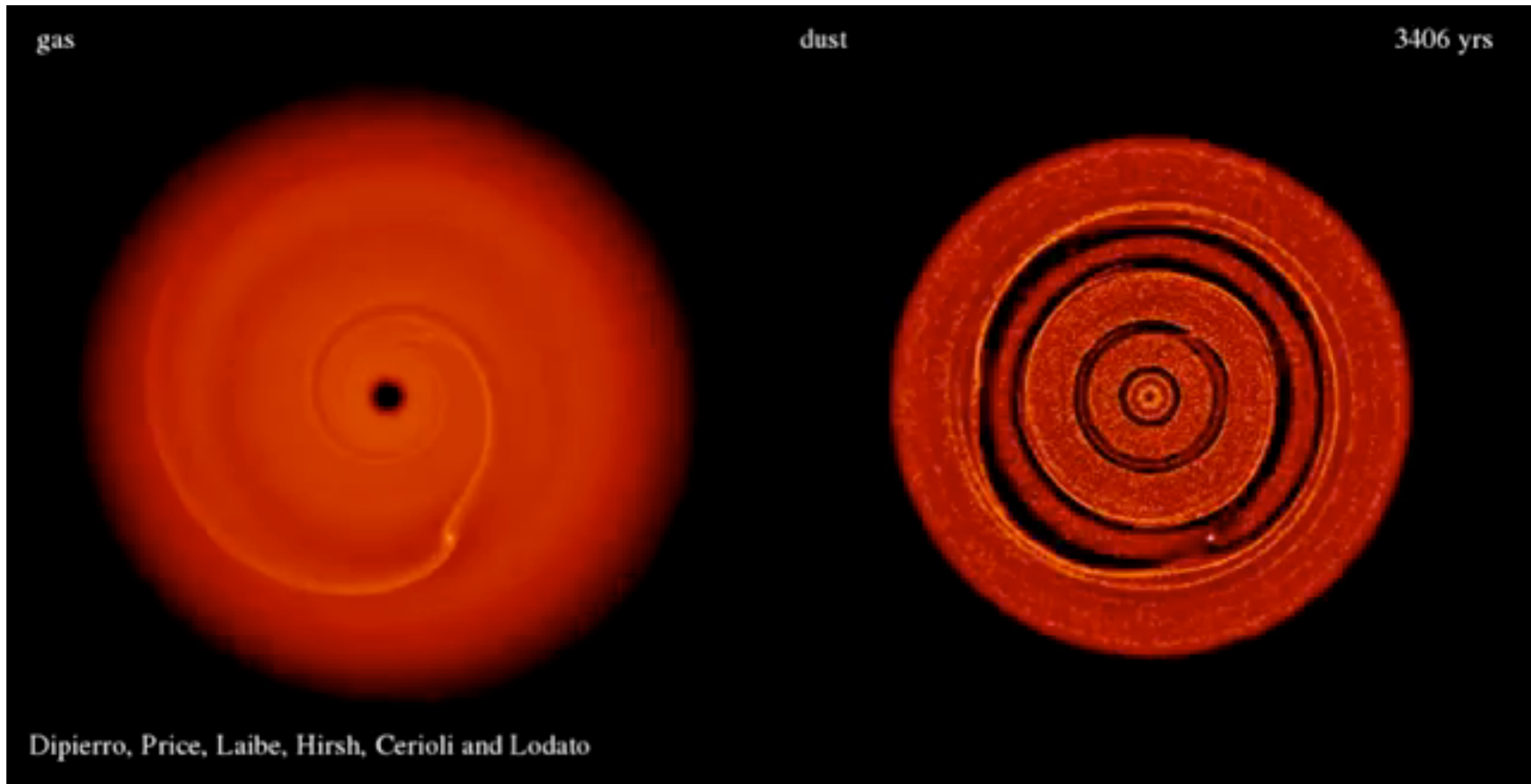
- Very young disks (like HL Tau) show possible signatures of gaps formed by large planets (some disks even show localized perturbations, which are very likely to be formed by such planets)
- Class II disks ($\gtrsim 1 - 3$ Myr in age) don't have enough dust mass to account for observed exoplanets
- Some meteorites are dated to have formed very early in the age of the Solar System (and they are fragments of collided planetesimals)
- **Planet formation mostly complete in $\lesssim 1$ Myr? Very open area of study!**

Growth by Giant Impacts



- Once planetary embryos form, they dynamically stir each other into eccentric orbits; over time (many Myr), they collide and grow into the terrestrial bodies
- Gas giants (if they've already formed) contribute to this process

Pressure bump caused by planets!



- If planet is sufficiently massive, gap is carved out
- Pressure bump exists on outer edge - traps pebbles

What mass is necessary to halt pebbles?

- Planetary core perturbs the gas density such that the azimuthal velocity is perturbed by

$$\Delta v_y \sim \frac{GM_p}{rH\Omega}$$

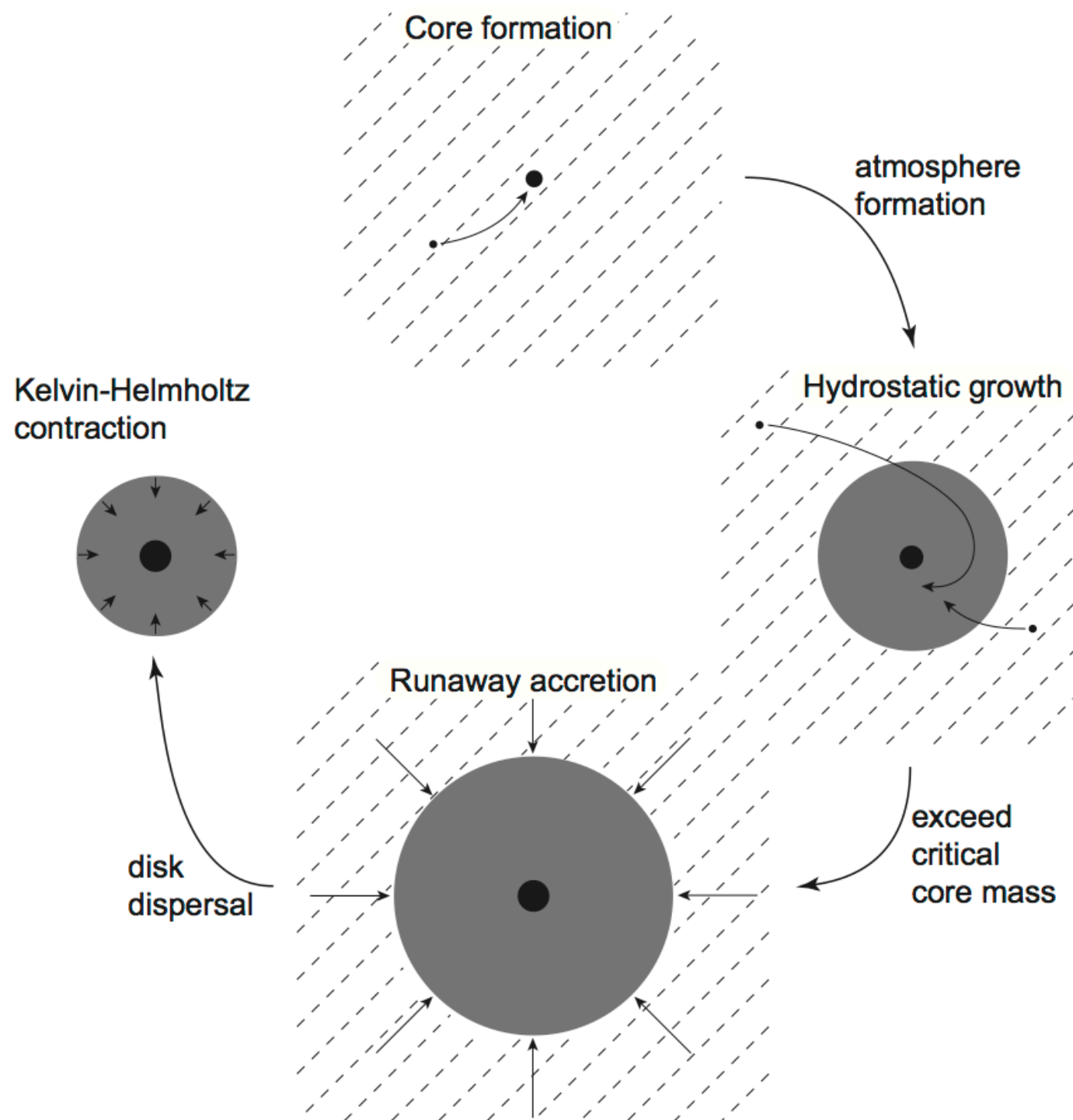
- If we set this equal to the deviation of the gas velocity from Keplerian,

$$\Delta v_y \sim \left(\frac{H}{r}\right)^2 \frac{\partial \ln P}{\partial \ln r} v_K, \text{ we can find } M_p \text{ such that the gas velocity is only Keplerian. Solving for } M_p \text{ we get:}$$

$$M_{\text{iso}} \sim \left(\frac{H}{r}\right)^3 \frac{\partial \ln P}{\partial \ln r} M_* \approx 20 \left(\frac{H/r}{0.05}\right)^3 M_{\oplus}$$

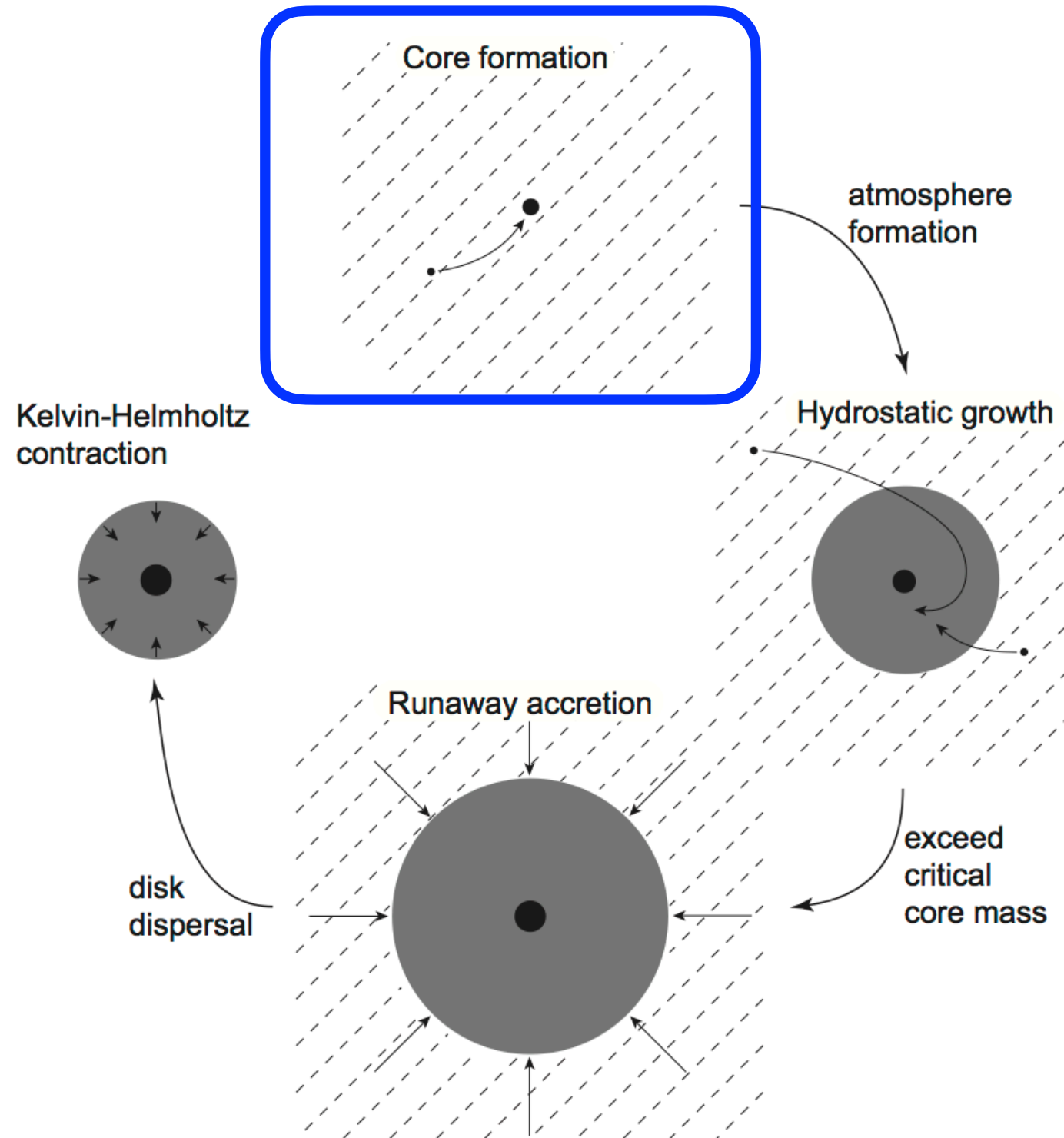
- The expression on the right hand side (approximating 20 Earth masses) comes from calibration to numerical simulations.
- More up-to-date formula calibrated to include turbulent gas (α) increases the isolation mass
- But the point is that this “pebble isolation mass” can be quite large compared to the planetesimal isolation mass**
- A gas giant core is necessary to halt pebble flow!***

Gas giant formation via core accretion



1. Core formation - pebble and/or planetesimal accretion; pebble accretion necessary for rapid formation (to get to the next stages)
2. Hydrostatic growth - gas slowly accretes; envelope adjusts and releases heat (deposited by planetesimal accretion and gravitational potential energy release)
3. Runaway accretion - envelope mass reaches core mass; planet can no longer maintain hydrostatic equilibrium - gas accretes extremely rapidly ($\tau_{\text{acc}} \sim 10^5 \text{ yr}$)
4. Termination of accretion - either gas in disk disperses or the planet carves out a gap and isolates itself
5. Continued Kelvin-Helmholtz contraction (Jupiter still undergoing this!)

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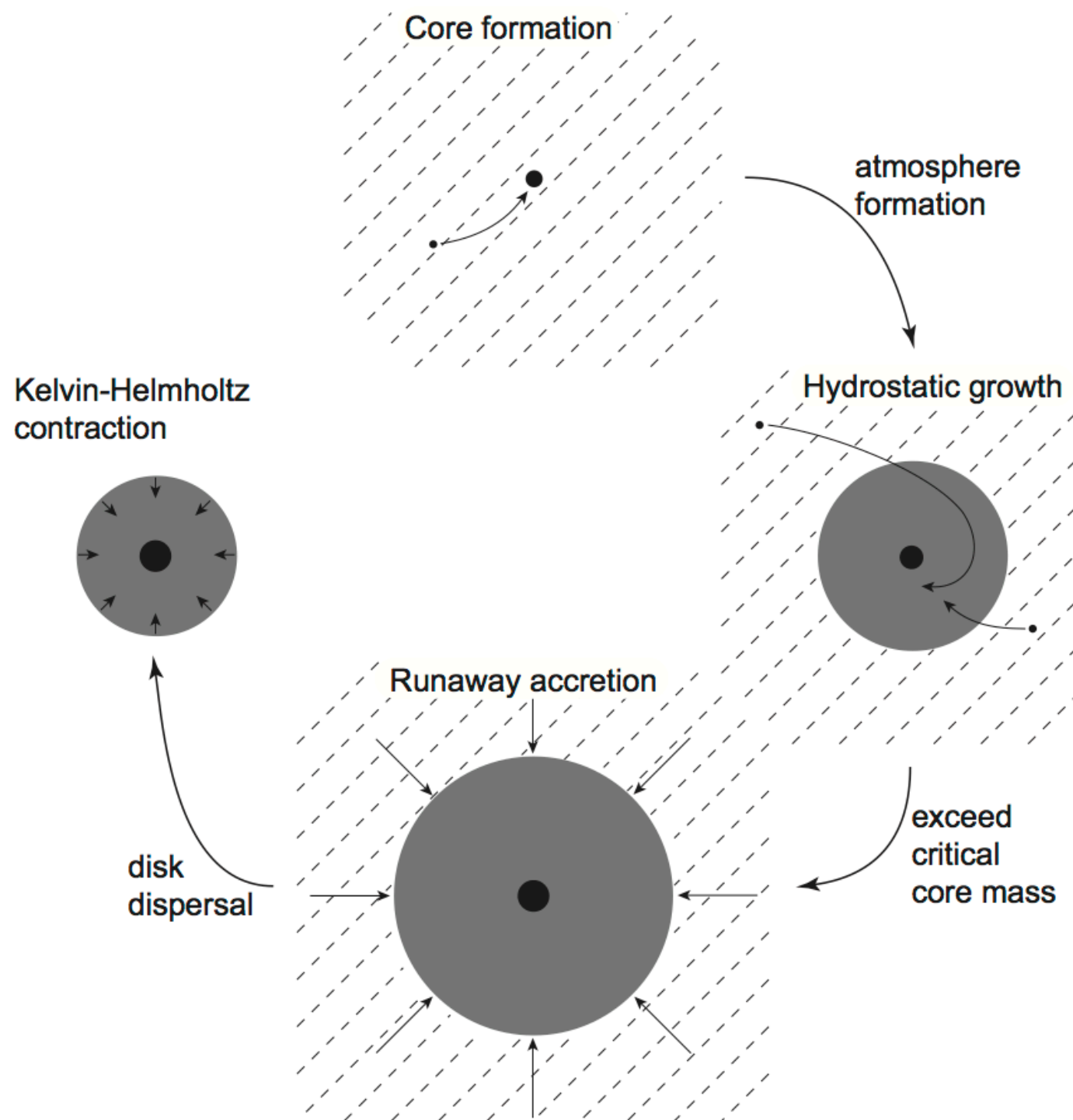
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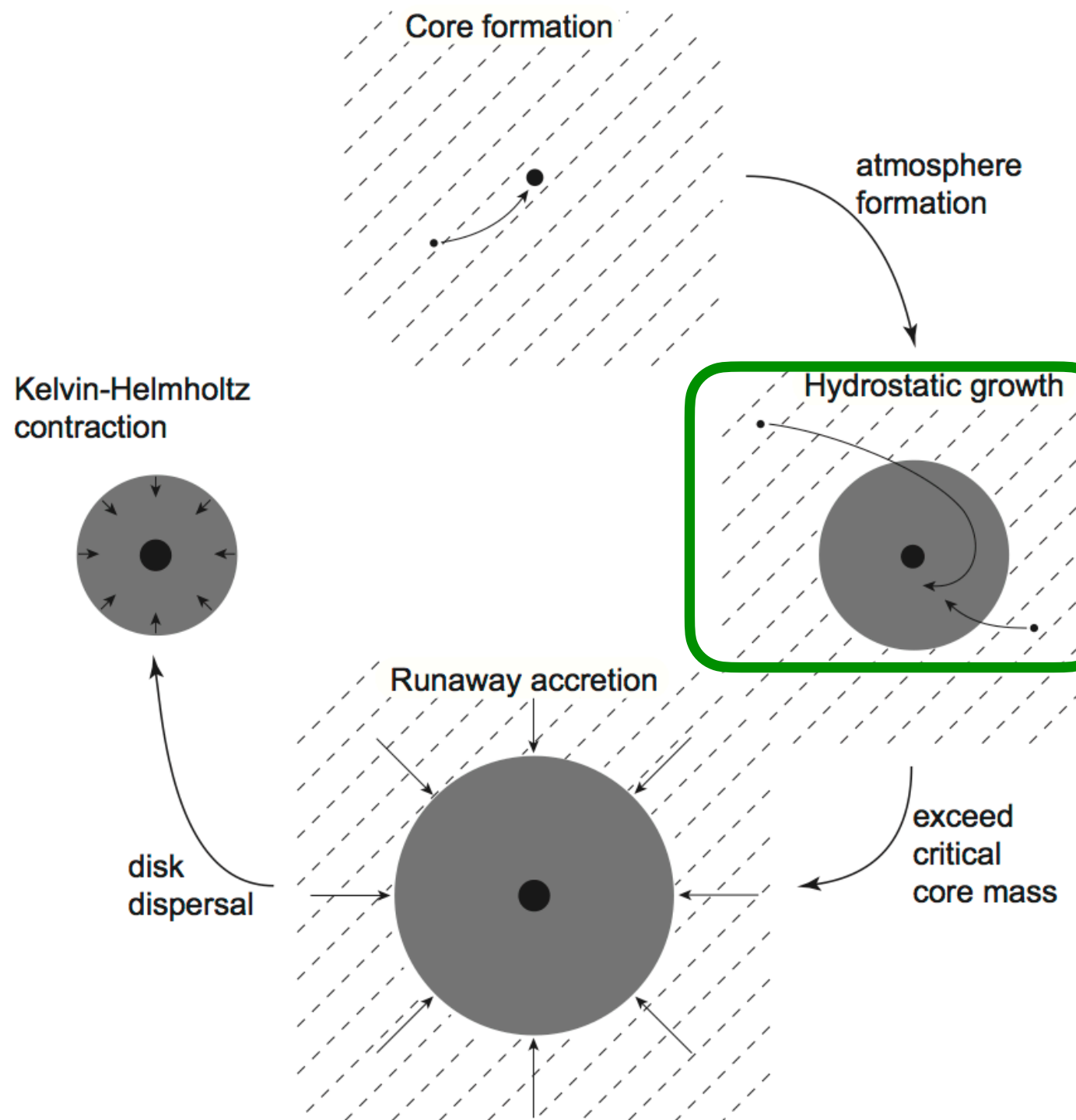
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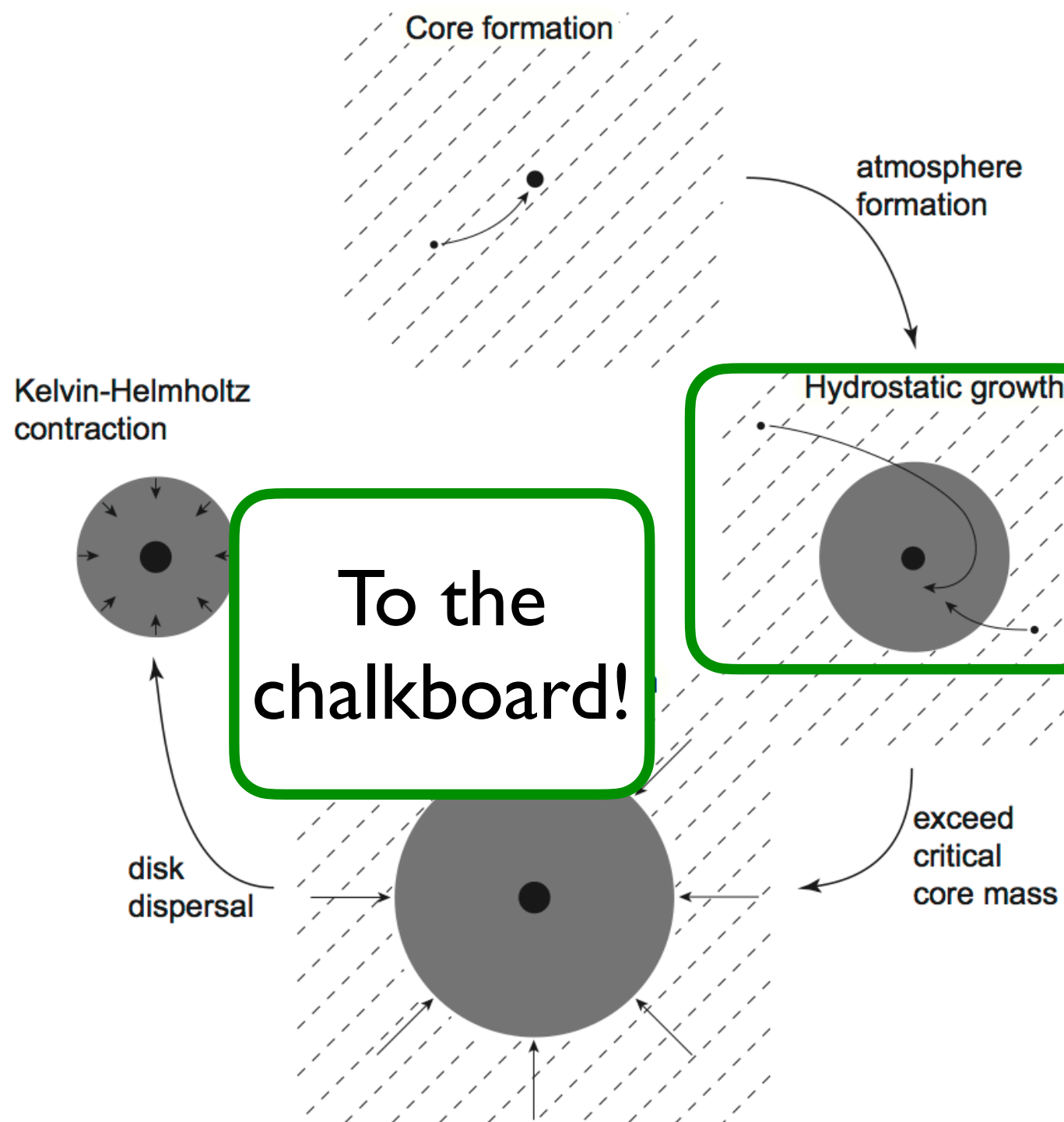
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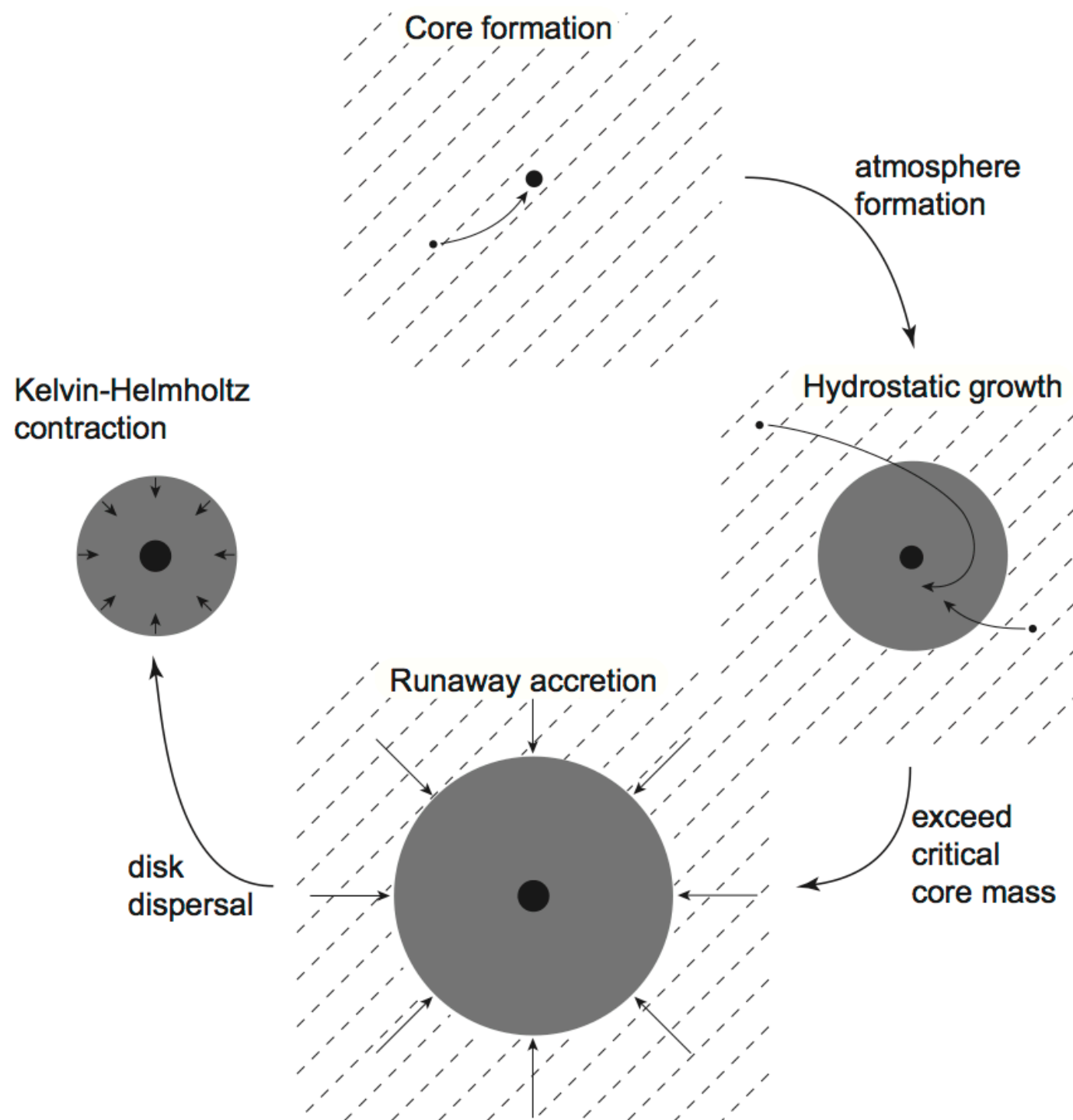
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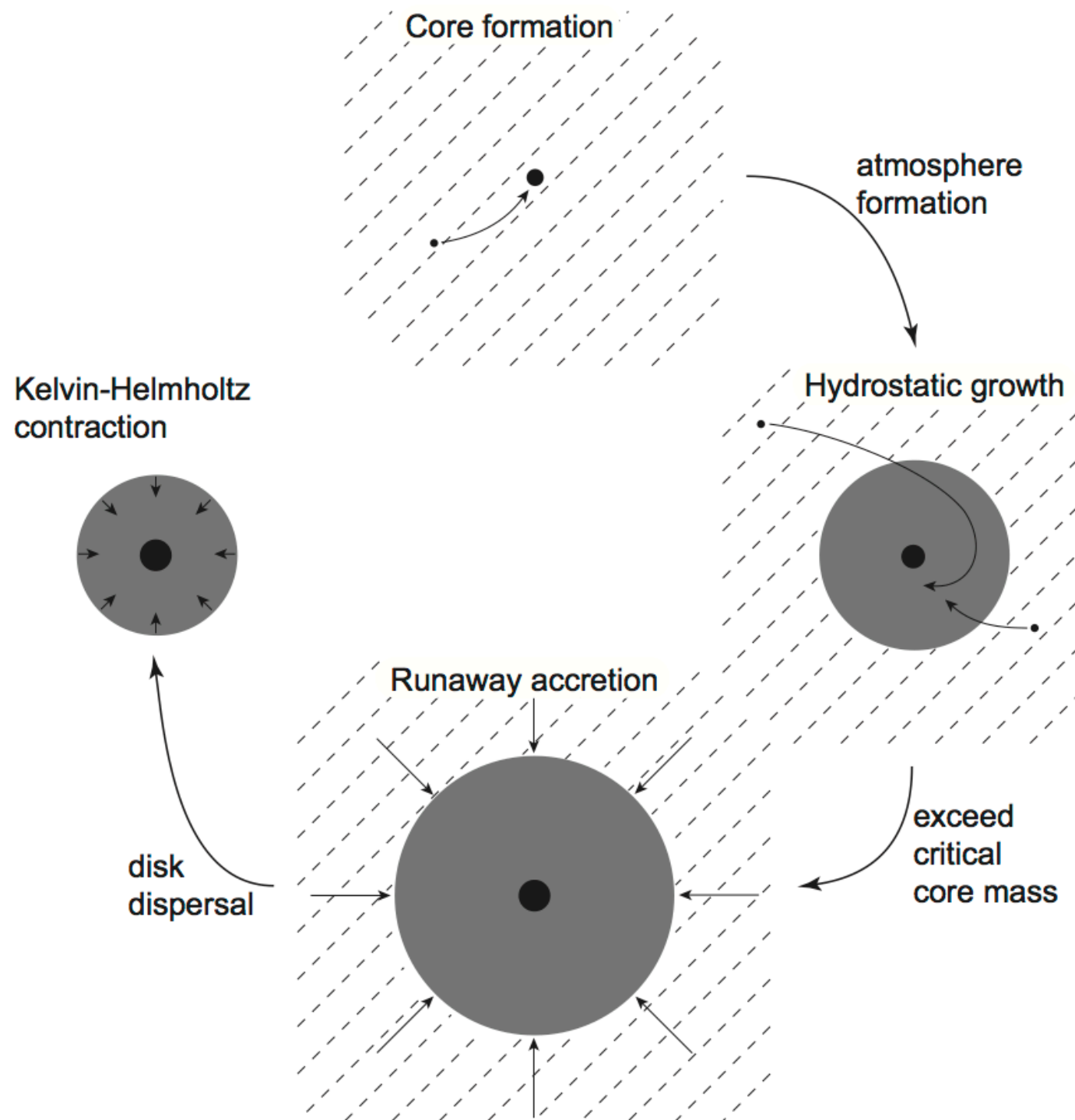
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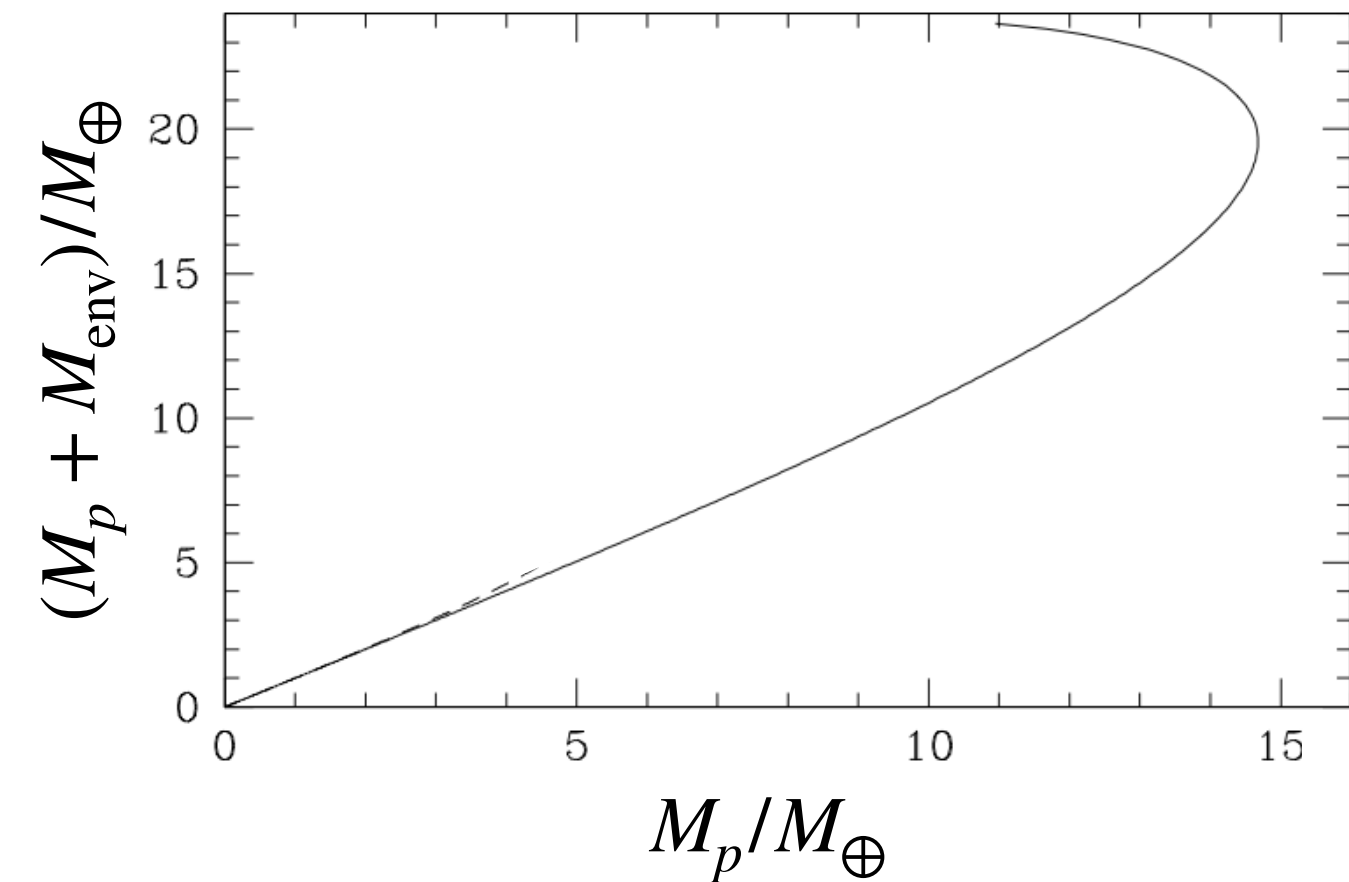
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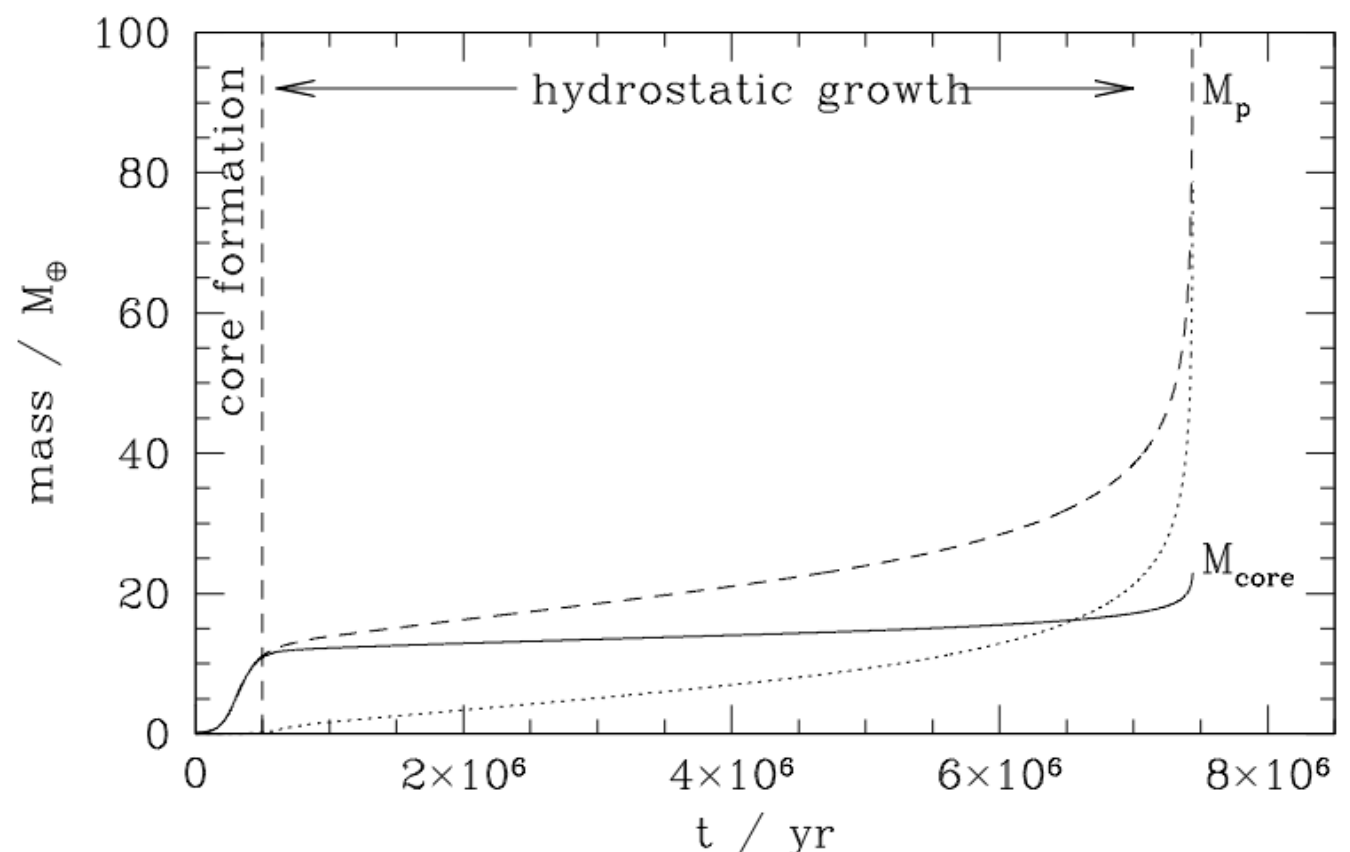
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Runaway gas accretion

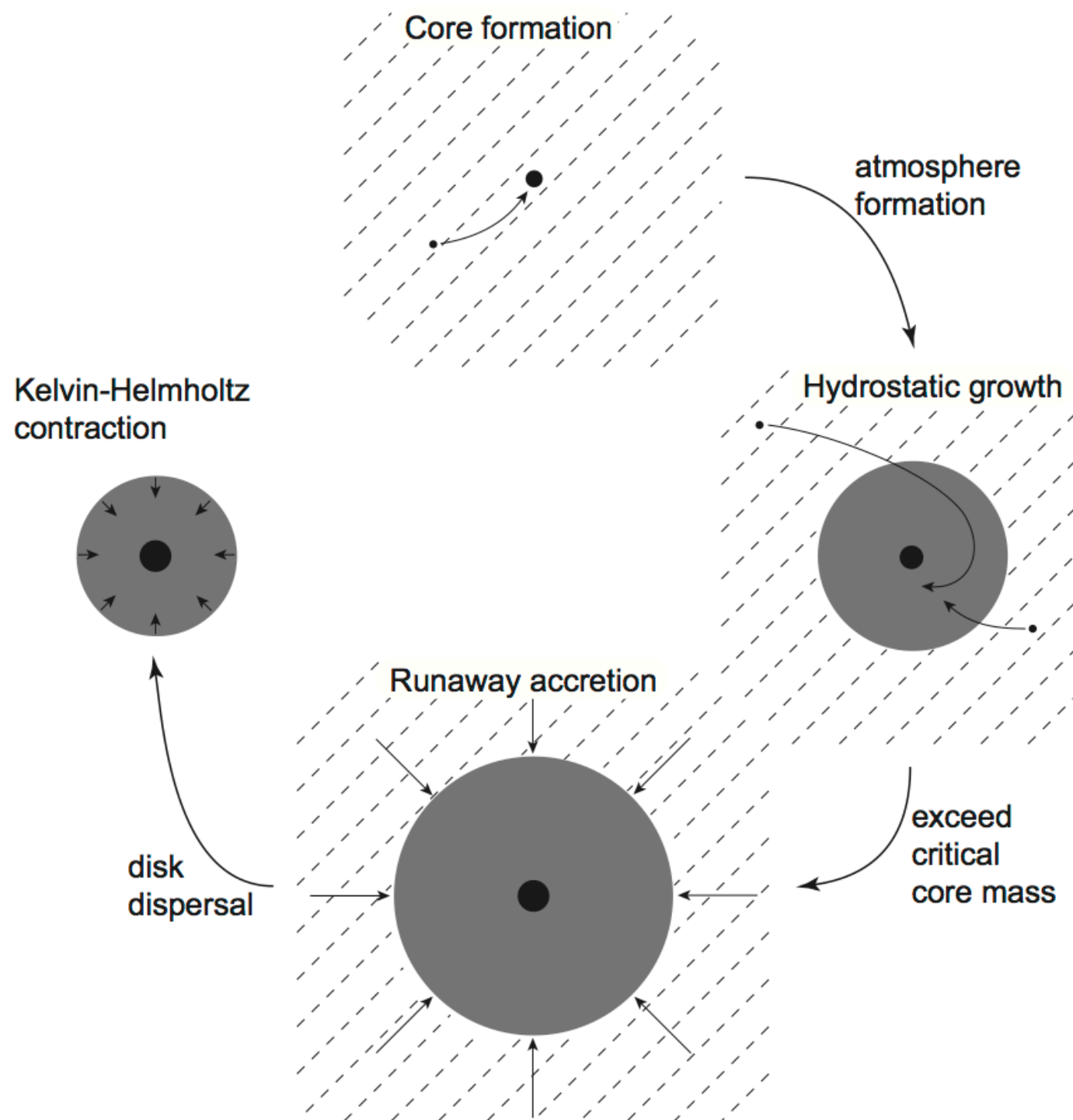


- After runaway accretion starts, it is *extremely fast*
- Physics of how it works: more mass is added to the envelope, but system cannot adjust to new mass (in terms of pressure and thermal transport) - more mass accretes and the system runs away
- What will stop it??

- Need numerical solution to radial structure equations
- Have to account for thermodynamic effects, such as radiative transport and convection (e.g., like in the Sun!)
- Find maximum core mass of $\sim 10 - 20 M_{\oplus}$ before runaway accretion

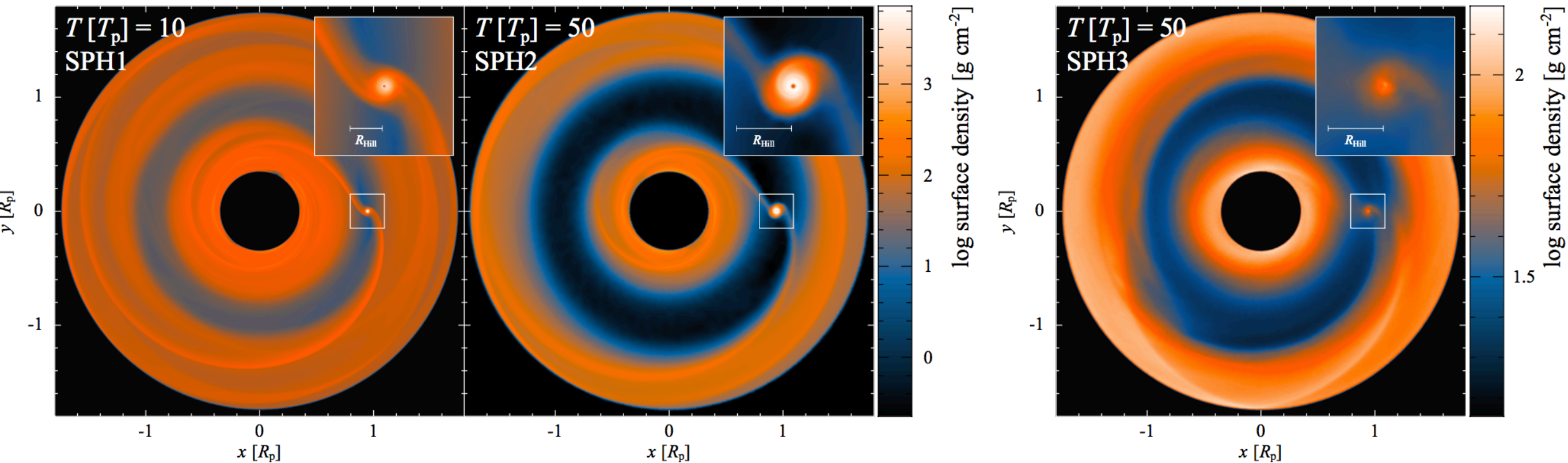


Core accretion model



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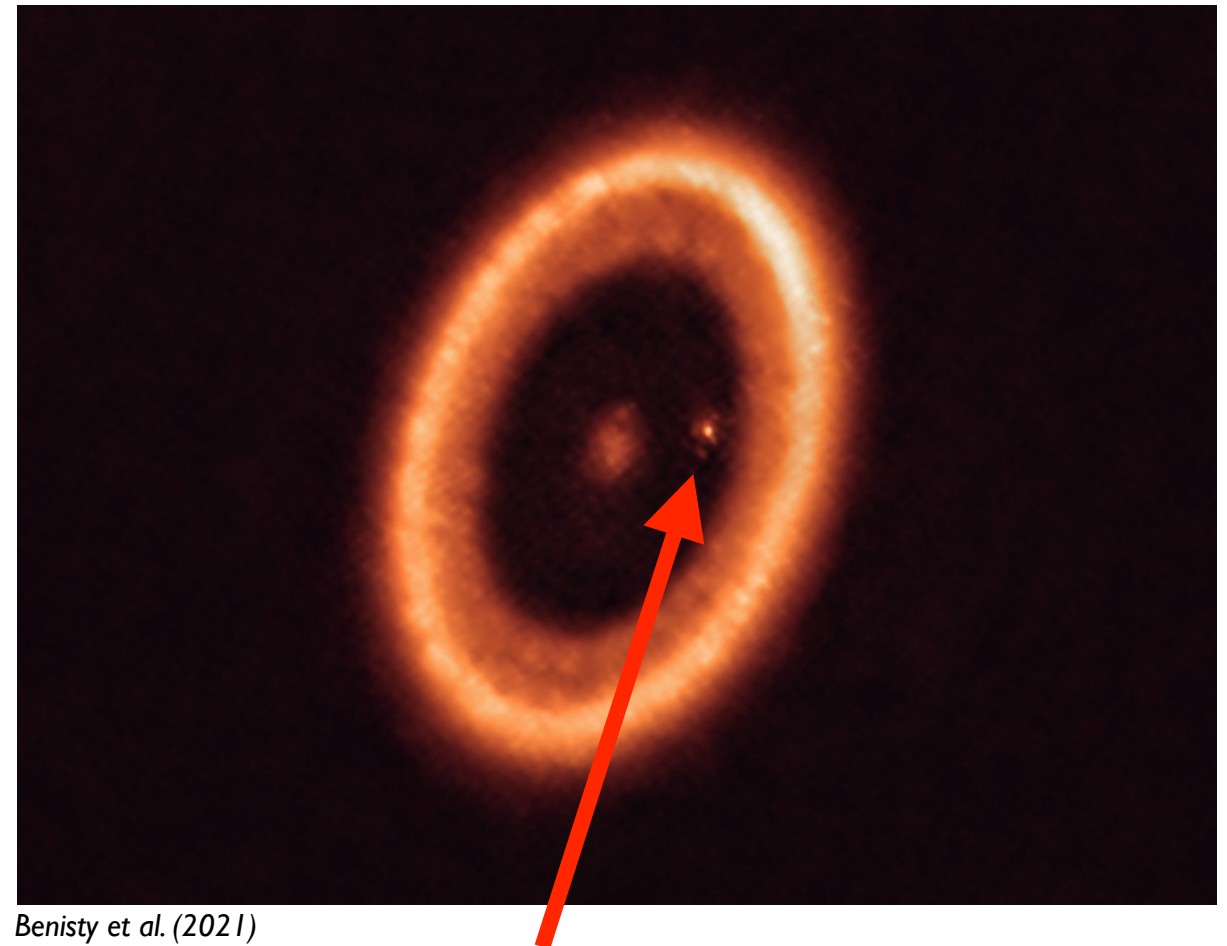
Circumplanetary Disks



- Part of the accretion process may involve a ***circumplanetary disk (CPD)***
- This occurs when there is sufficient angular momentum such that the accreting gas cannot simply fall onto the forming giant
- ***Still very open area of study: the processes by which gas flow through the protoplanetary disk to the CPD and ultimately accrete onto the forming giant - involves large-scale numerical simulations***

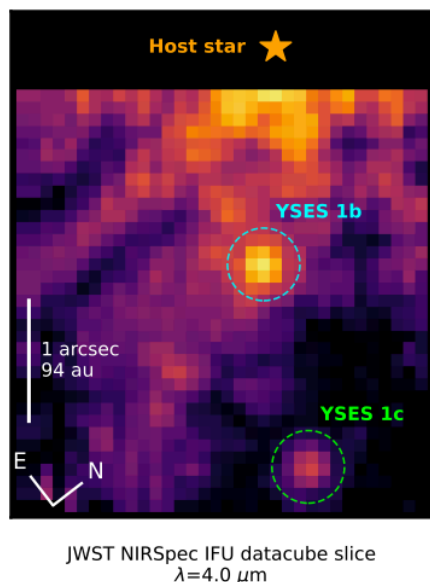
CPD observations

- Problem: there are far fewer CPDs observed than expected!
 - Still open area of study - not yet sure why there are so few
 - Maybe a more spherical accretion flow is more common?
- But two confirmed cases: PDS70c and YSES-1b (more on the way)

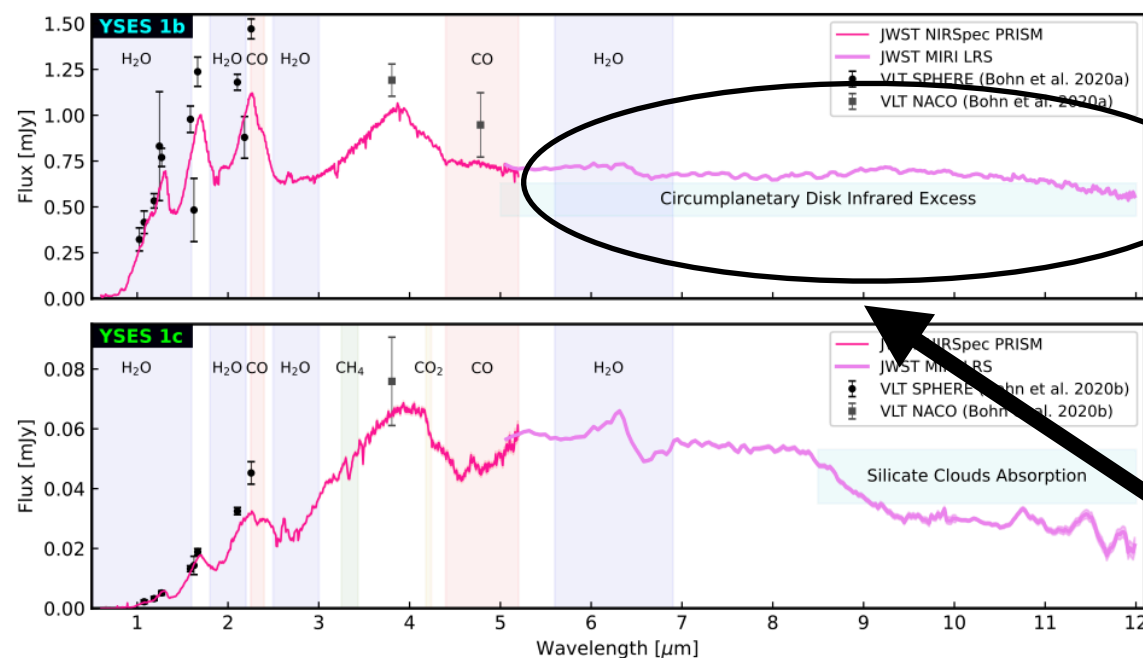


Benisty et al. (2021)

Thermal emission due to dust in a CPD (notice larger disk with a huge gap, possibly carved out by planets!)

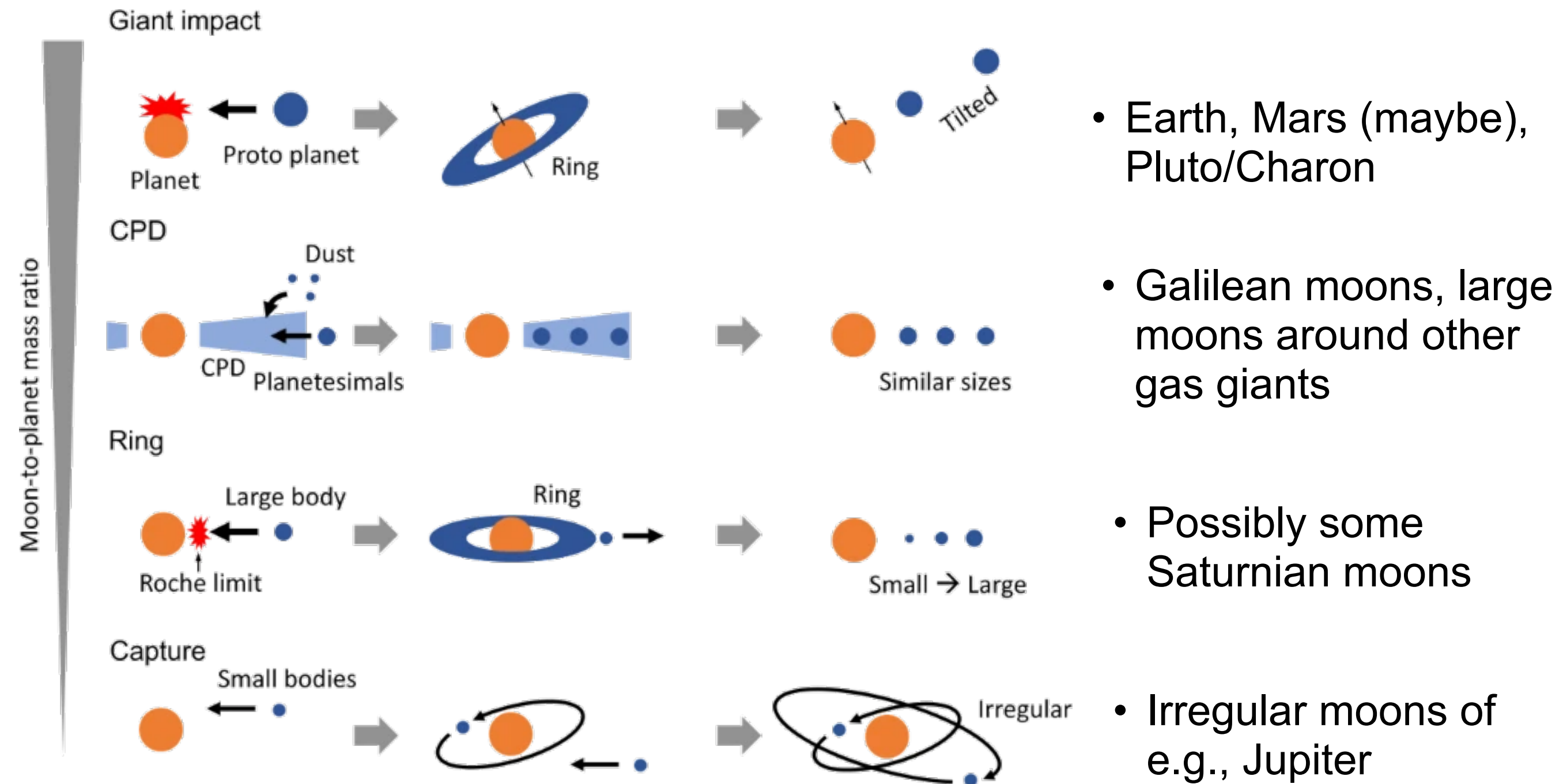


Hoch et al. (2025), Nature

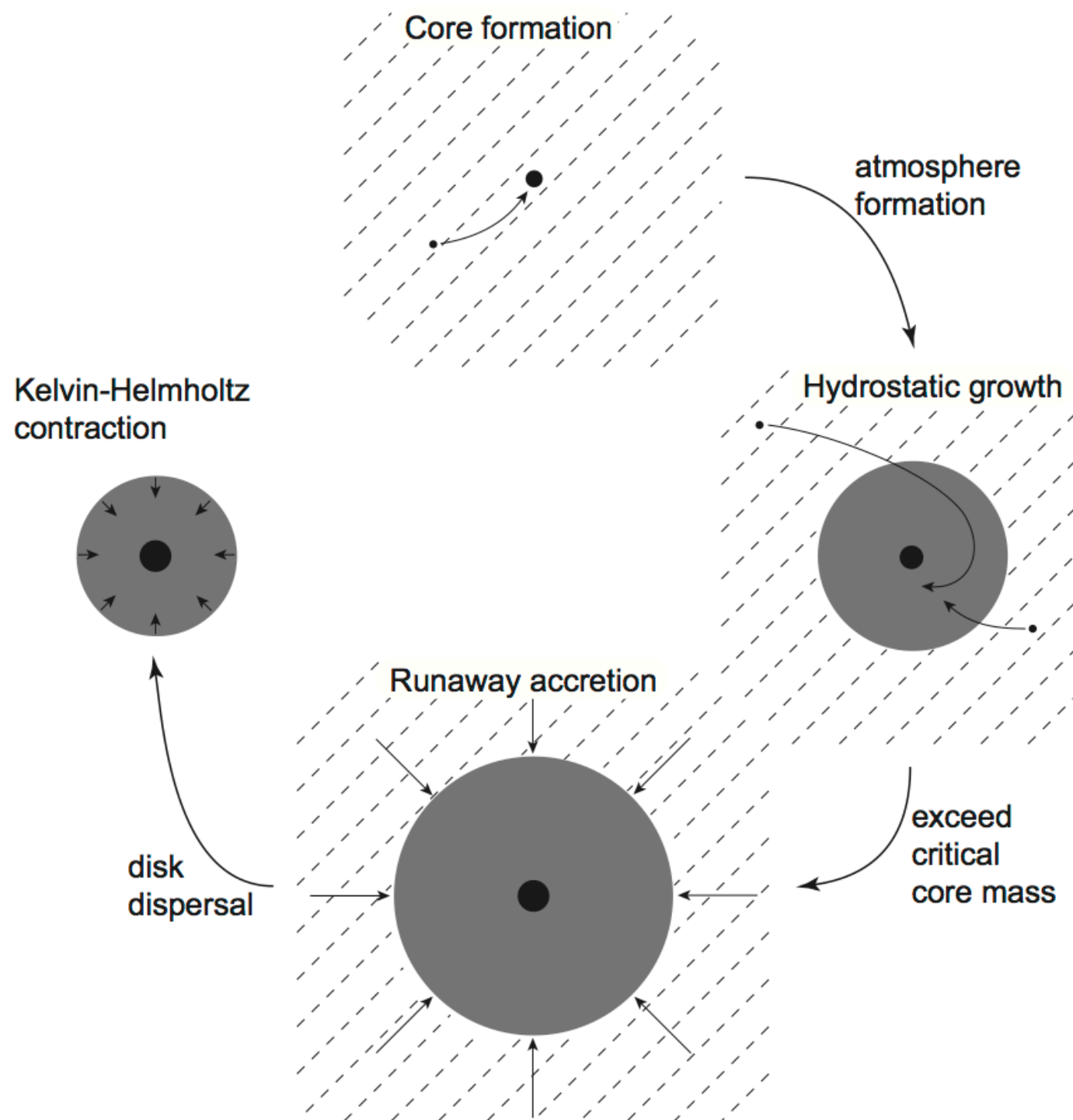


Infrared excess associated with disk

Formation of moons



Core accretion model



1. Core formation - pebble and/or planetesimal accretion; pebble accretion necessary for rapid formation (to get to the next stages)
2. Hydrostatic growth - gas slowly accretes; envelope adjusts and releases heat (deposited by planetesimal accretion and gravitational potential energy release)
3. Runaway accretion - envelope mass reaches core mass; planet can no longer maintain hydrostatic equilibrium - gas accretes extremely rapidly ($\tau_{\text{acc}} \sim 10^5 \text{ yr}$)
4. Termination of accretion - either gas in disk disperses or the planet carves out a gap and isolates itself
5. Continued Kelvin-Helmholtz contraction (Jupiter still undergoing this!)

Gravitational instability model (reviewed)

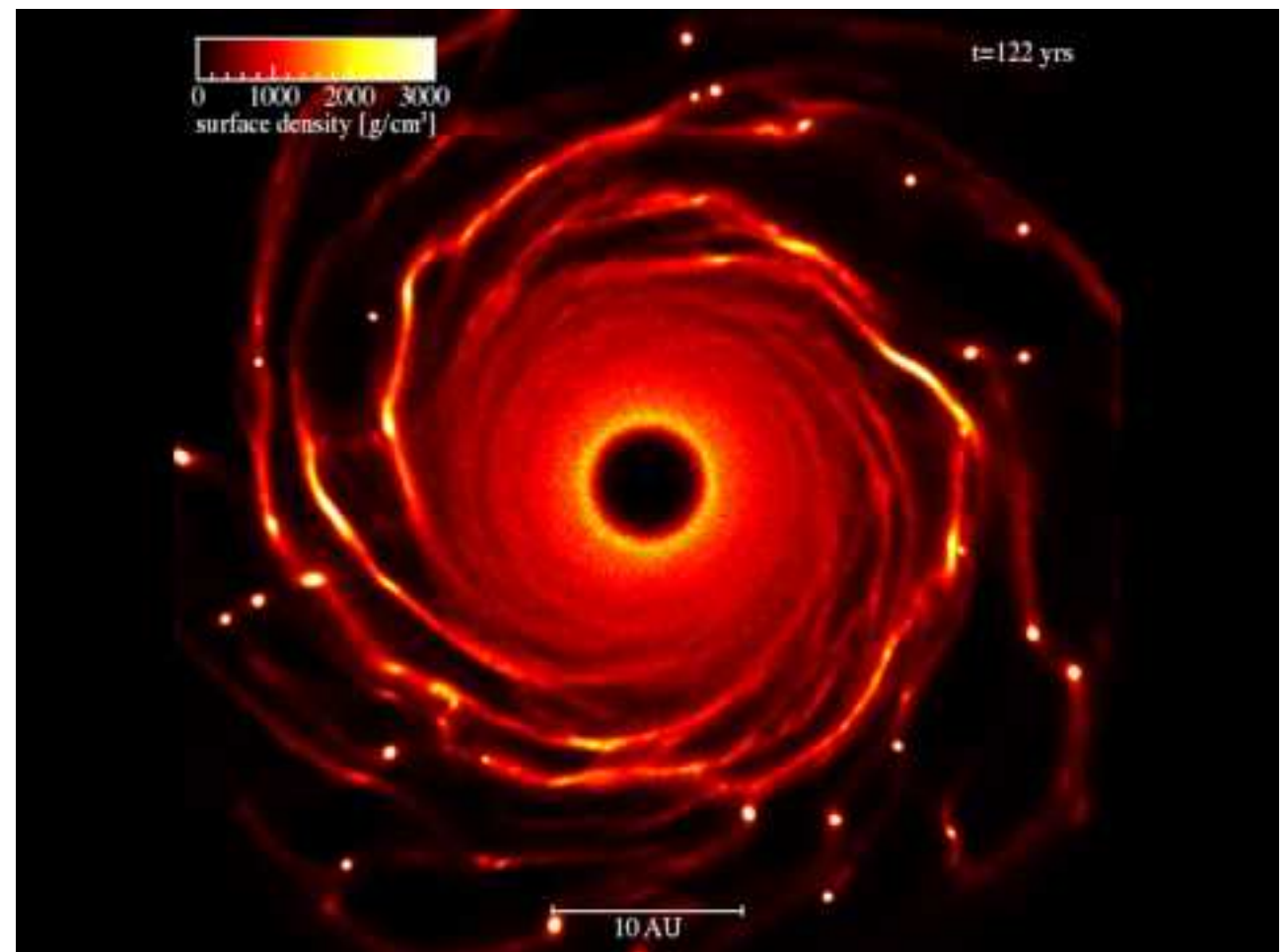
- Some disks can be gravitationally unstable as we have already seen.
- Condition for this:
- **Rapid cooling (on dynamical timescale!!) is necessary to overcome negative feedback mechanism** — to the chalkboard to see why

$$Q \equiv \frac{c_s \Omega}{\pi G \Sigma} \lesssim 1$$

- **But also needs to cool sufficiently rapidly**
- **Necessary cooling timescale:**

$$\tau_{\text{cool}} \approx 3\Omega^{-1}$$

(determined by numerical simulations)



Daniel Price - Gravitationally unstable disk that fragments into bound objects