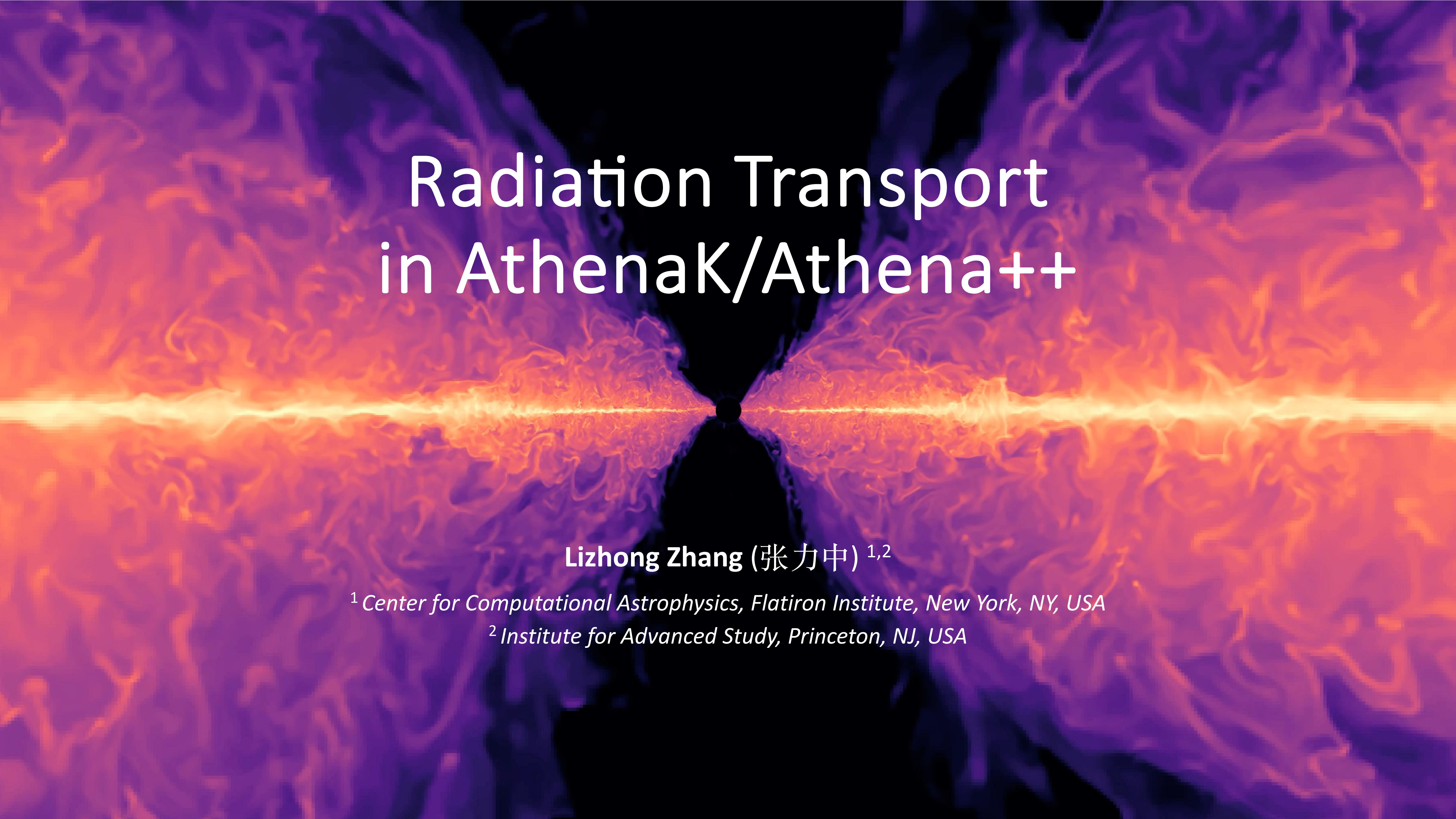


Radiation Transport in AthenaK/Athena++

Lizhong Zhang (张力中) [1,2]

[1] *Center for Computational Astrophysics, Flatiron Institute, New York, NY, USA*

[2] *Institute for Advanced Study, Princeton, NJ, USA*

Covariant Radiative Transfer

Radiation Transport in GR:

$$\partial_0\left(\hat{n}^0\hat{n}_0\hat{I}_\nu\right) + \underbrace{\frac{1}{\sqrt{-g}}\partial_j\left(\sqrt{-g}\,\hat{n}^j\hat{n}_0\hat{I}_\nu\right)}_{\text{spatial flux}} + \underbrace{\partial_{\hat{\nu}}\left(\hat{n}^{\hat{\nu}}\hat{n}_0\hat{I}_\nu\right)}_{\text{frequency flux}} + \underbrace{\frac{1}{\sin\hat{\zeta}}\partial_{\hat{\zeta}}\left(\sin\hat{\zeta}\,\hat{n}^{\zeta}\hat{n}_0\hat{I}_\nu\right) + \partial_{\hat{\psi}}\left(\hat{n}^{\hat{\psi}}\hat{n}_0\hat{I}_\nu\right)}_{\text{angular flux}} = \underbrace{\hat{n}_0\left(\hat{j}_\nu - \hat{\chi}_\nu\hat{I}_\nu\right)}_{\text{source terms}}$$

Time Evolution | Spatial Propagation | Gravitational Redshift/Blueshift | Light Bending Effect | Radiation-Gas Coupling

See Davis & Gammie (2020)

$\sqrt{-g} = 1$ in Cartesian Kerr-Schild Coordinates

$\hat{I}_\nu$: Radiation Intensity (erg cm^{-2} s^{-1} sr $^{-1}$ Hz^{-1}) $\rightarrow$ $\boxed{\dfrac{\text{Energy Flux}}{\text{Solid angle} \cdot \text{Frequency}}}$

Grey Radiation Transfer in AthenaK

Frequency-Integrated Radiation Transport:

$$\partial_0\left(\hat{n}^0\hat{n}_0\hat{I}\right) + \underbrace{\partial_j\left(\hat{n}^j\hat{n}_0\hat{I}\right)}_{\text{spatial flux}} + \underbrace{\frac{1}{\sin\hat{\zeta}}\partial_{\hat{\zeta}}\left(\sin\hat{\zeta}\,\hat{n}^{\zeta}\hat{n}_0\hat{I}\right) + \partial_{\hat{\psi}}\left(\hat{n}^{\hat{\psi}}\hat{n}_0\hat{I}\right)}_{\text{angular flux}} = \underbrace{\hat{n}_0\left(\hat{j} - \hat{\chi}\hat{I}\right)}_{\text{source terms}}$$

White et al. (2023)

Operator Splitting:

Solving advection in the tetrad frame

$$\partial_0\left(\hat{n}^0\hat{n}_0\hat{I}\right) + \partial_j\left(\hat{n}^j\hat{n}_0\hat{I}\right) + \frac{1}{\sin\hat{\zeta}}\partial_{\hat{\zeta}}\left(\sin\hat{\zeta}\,\hat{n}^{\zeta}\hat{n}_0\hat{I}\right) + \partial_{\hat{\psi}}\left(\hat{n}^{\hat{\psi}}\hat{n}_0\hat{I}\right) = 0$$

Solving source terms in the fluid frame

$$\partial_0\left(\hat{n}^0\hat{n}_0\hat{I}\right) = \hat{n}_0\left(\hat{j} - \hat{\chi}\hat{I}\right)$$

Radiation Transfer

A. Demonstrate how radiation transfer equation works in energy and momentum spaces

B. Demonstrate how the source terms are solved in solid angle discretization framework

Let's start from a simpler version:

$$\frac{1}{c}\frac{\partial I_\nu}{\partial t} + \hat{n} \cdot \boldsymbol{\nabla} I_\nu = \eta_\nu - \chi_\nu I_\nu$$

Blackboard derivation now …

Upcoming Radiation Modules

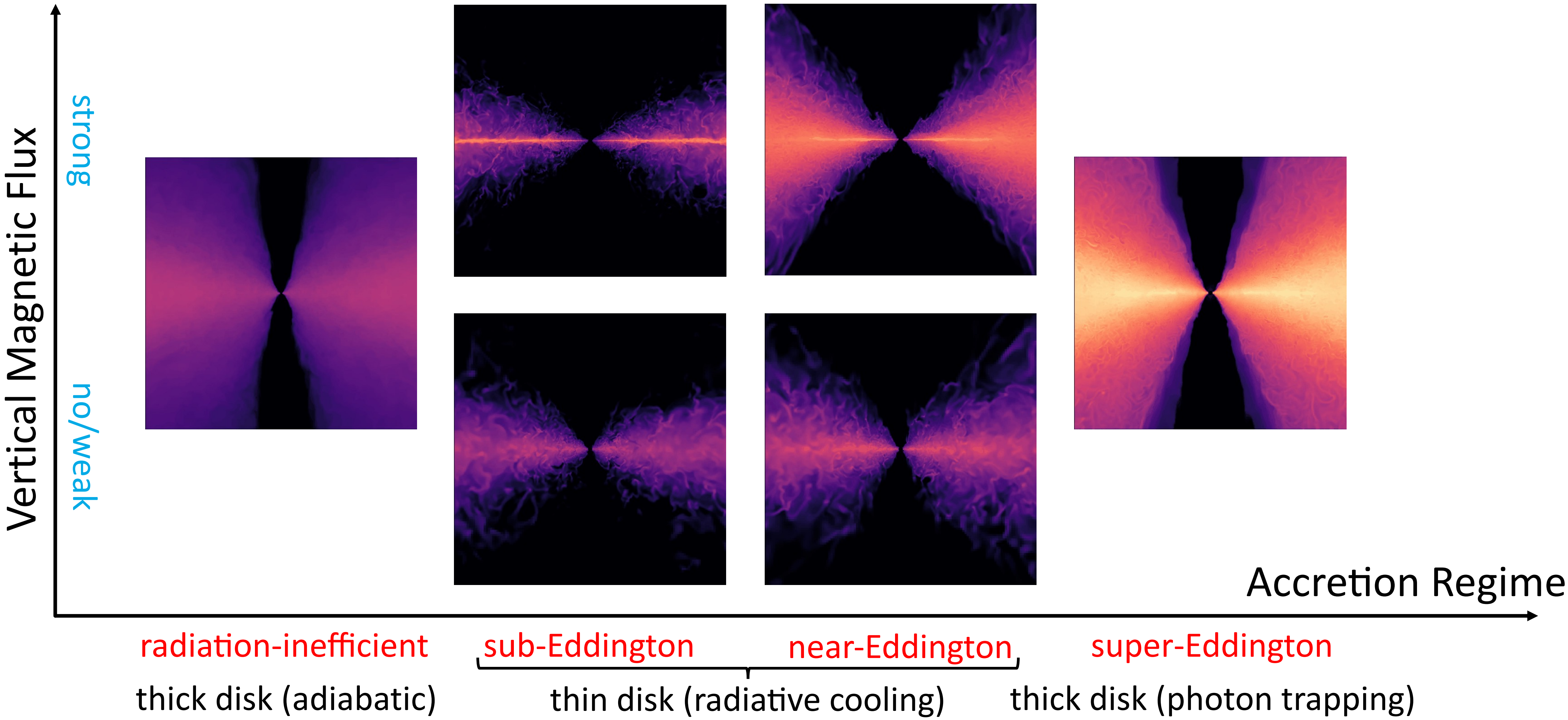
A. Multi-Frequency Group in Radiation Transport:

$$\partial_0\left(\hat{n}^0\hat{n}_0\hat{I}_\nu\right) + \underbrace{\frac{1}{\sqrt{-g}}\partial_j\left(\sqrt{-g}\,\hat{n}^j\hat{n}_0\hat{I}_\nu\right)}_{\text{spatial flux}} + \underbrace{\partial_{\hat{\nu}}\left(\hat{n}^{\hat{\nu}}\hat{n}_0\hat{I}_\nu\right)}_{\text{frequency flux}} + \underbrace{\frac{1}{\sin\hat{\zeta}}\partial_{\hat{\zeta}}\left(\sin\hat{\zeta}\,\hat{n}^{\zeta}\hat{n}_0\hat{I}_\nu\right) + \partial_{\hat{\psi}}\left(\hat{n}^{\hat{\psi}}\hat{n}_0\hat{I}_\nu\right)}_{\text{angular flux}} = \underbrace{\hat{n}_0\left(\hat{j}_\nu - \hat{\chi}_\nu\hat{I}_\nu\right)}_{\text{source terms}}$$

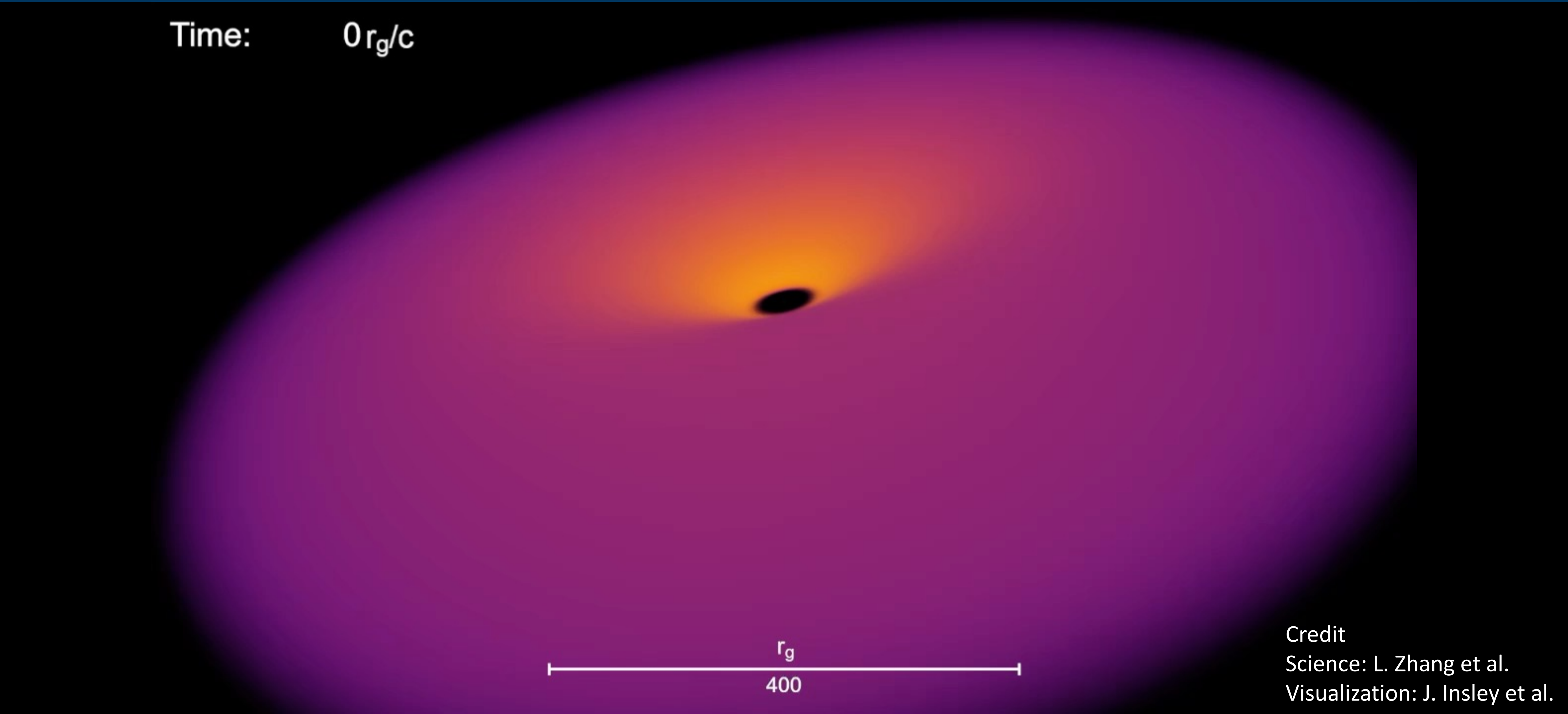
B. Polarization in Radiation Transport:

$$\frac{1}{c}\frac{\partial}{\partial t}\begin{pmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{pmatrix} + \hat{\boldsymbol{n}}\cdot\boldsymbol{\nabla}\begin{pmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{pmatrix} = \underbrace{\begin{pmatrix} j_\nu \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{\substack{\text{thermal}\\\text{emission}}} \underbrace{- \chi_{\nu,a}\begin{pmatrix} I_\nu \\ Q_\nu \\ U_\nu \\ V_\nu \end{pmatrix}}_{\substack{\text{thermal}\\\text{absorption}}} + \underbrace{\boldsymbol{\Lambda}_{\text{scat,attenuation}}}_{\substack{\text{scattering}\\\text{attenuation}}} + \underbrace{\boldsymbol{\Lambda}_{\text{scat,emission}}}_{\substack{\text{scattering}\\\text{emission}}}$$

Radiation-Dominated Accretion onto Black Holes

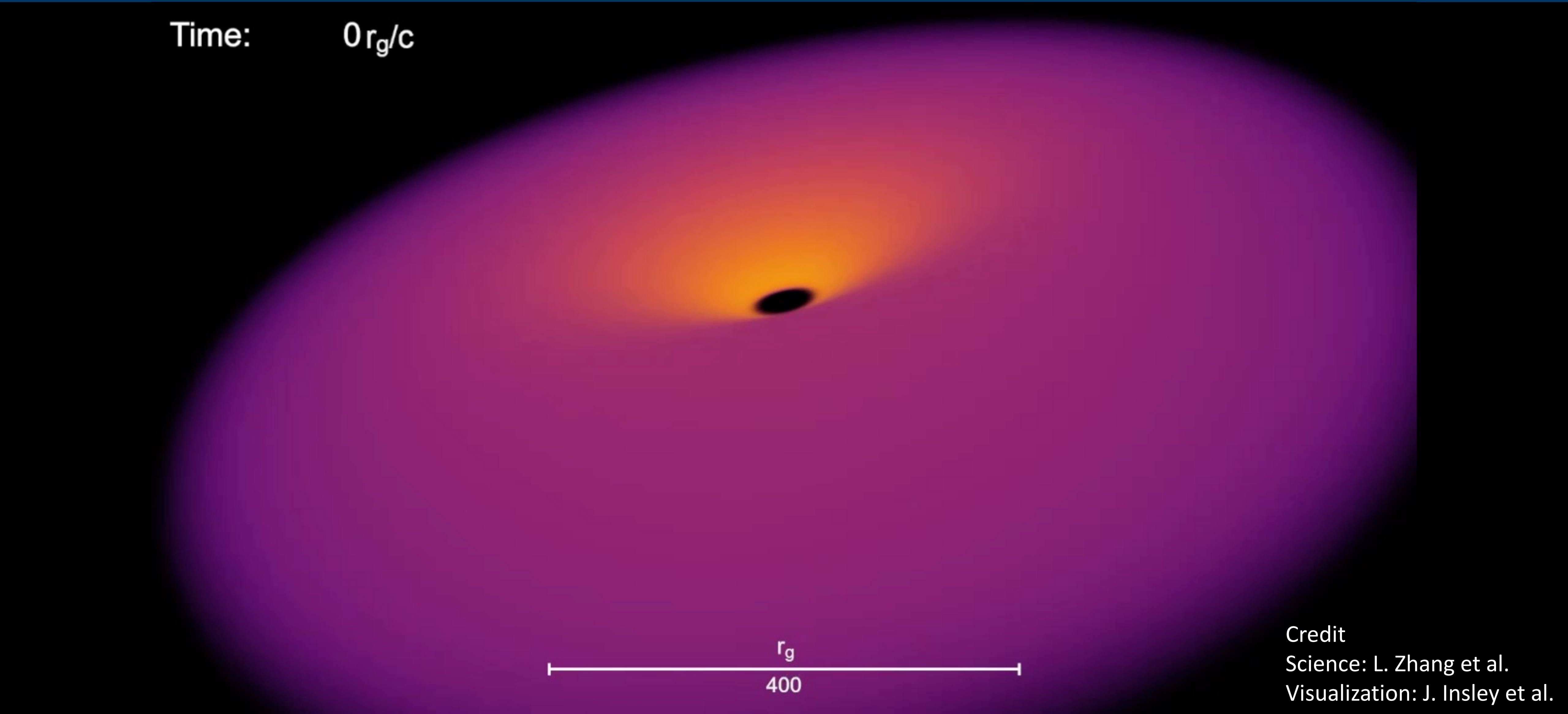


3D Radiation Energy Density



Credit
Science: L. Zhang et al.
Visualization: J. Insley et al.

Relativistic Jet



Credit
Science: L. Zhang et al.
Visualization: J. Insley et al.